1996

Closed loop process identification for multi-variable systems

Hemant Jha
Lehigh University

Hemant, Jha

Closed Loop Process Identification for Multi-Variable Systems

June 2, 1996

CLOSED LOOP PROCESS IDENTIFICATION FOR MULTI-VARIABLE SYSTEMS

by

Hemant Jha

A Thesis
Presented to the Graduate and Research Committee
of Lehigh University
in Candidacy for the Degree of
Master of Science
in
Chemical Engineering

Lehigh University
May, 1996

This thesis is accepted in partial fulfillment of the requirements for the degree of Master of Science.

May 6, 1996

(Date)

Prof. Christos Georgakis

Prof. Dennis Hess

To my parents ......

माना की इस ज़मीन को न गुल्ज़ार कर सके
कुछ खार कम तो कर गये, गुज़रे जिधर से हम

Preface

Scores of literature have been published in the area of process identification. Most of them work fine for simple systems. But the real life problems are quite complex and multivariable in nature. There are, however, very few contributions which lead to algorithms which really work in practice. So from the viewpoint of practitioner one can say.

"Yes everything seems to work well in theory, but not in practice."

From the theoretician point of view this can be answered in two ways.

" The implementation of the underlying theory is not perfect and one needs to develop effective implementation method"

or

"The present body of knowledge is not sufficient to solve the problem efficiently, hence there is a need to develop a new method"

I tried to address the problem form the latter prospective. Bridging the gap between the theory and practice of process identification has been the primary issue of my research. I feel that theory which can be translated into practice is far better than its counterpart which fails to cross the boundary.

Some of the chapters presented in this thesis would be difficult to grasp for the first time reader or a novice in the area of process identification. I would like to apologize for that, but would like to point out, "A step in a right direction is always better that no step at all".

May 7th, '96

Hemant Jha
Lehigh University

Acknowledgments

First of all, I would like to thank my advisor Professor Christos Georgakis for his guidance and support during the course of my research. His encouragement and positive critic made me look at things differently and inspired me to surpass myself. The interaction with him helped me to enriched myself not only on professional but also on social level, and for that I am forever greatful. I would like to thank Phani B. Sistu his guidance and insight during the initial part of this project.

I am indebted to my parents for their continuous support and love. Their love and affection provided me the strength and determination and helped me to accomplish the things I have done. The same is true for my sisters, Anjana and Ritu , whose remarkable friendship and company kept me going.

I am specially greatful to my dearest friend Abhijit Ray for his advises and who's tremendous friendship I will cherish for ever. And to Gautam , Radhakrishnan, Aman, Manish, Deepak, Sanjay, Rupak and Partha for their support and inspiration. My thanks to my roommates, Sami, Indro, Satish and Jaleel, for giving me constant support and encouragement. Special thanks to Ishita and Sami who taught me *Sher-o-Sayari*, which kept me going through the difficult part of my project.

Finally want to thank all my colleagues at CPMC. Specially Lokesh Kalra, who helped me to learn some of the basics of controls and Wenfu Ku, who taught me some neat matlab tricks. And to everyone else, Tim Elliott, Vince Liotta, Joel Cantrell, Bill Doctor, Phil Lyman, Matt Watson, Stella Bezergianni and Fransisco, for making my stay a memorable and an enjoyable one.

Thanks to all of them and those I forgot.

Contents

List of Tables

List of Figures

Abstract

The availability of the process model is the first and the most important step in the design of a model based controller. The dynamic model can be obtained from a process operating in open or closed-loop configuration. Closed-loop identification is beneficial not only from the economic viewpoint but also from the perspective of the controller's design. Selecting the injection point for the identification input signal, affect the frequency accuracy of the derived model. Prediction Error Method (PEM) and Subspace methods are the two major classes of techniques which are examined here. The classical PEM methods, like direct identification, work well for data having low noise to signal ratio. The accuracy of the identified model deteriorates as the noise contribution in the signal is increased. The Two Step Method overcomes this drawback by breaking the problem in two open-loop identification step. It is indicated that the existing controller tuning plays a vital role in deciding the location of the external signal. The signal is injected such that the corresponding input-output sensitivity function is flat in the desired frequency range. This leads to lower bias in the identified model.

For multivariable systems the number of parameters to be identified for PEM is quite large and the convergence to the global minimum is difficult to achieve because it involves the solution to a nonlinear optimization problem. Subspace techniques are non-parametric and non-iterative in nature and hence are suitable in the identification of complex multivariable systems. They use advanced linear algebra concepts, like matrix projection, singular value decomposition and QR factorization for an effective implementation. Matrix projections form the basis for subspace algorithms and hence are examined in some detail. The closed-loop problem is solved in a

joint input-output manner. The open-loop subspace technique is applied to identify the overall closed-loop transfer function between the combined output (consisting of plant inputs and outputs) and the external signal from which the information about the plant dynamics can be obtained by suitable matrix manipulation. A symetric view of the closed-loop configuration is used to prove that defining the combined output in terms of controller input and output variables results in the identification of the controller transfer function. The required matrix manipulations are explained in terms of the state space description of the overall closed-loop system. This closed-loop algorithm is simplified further and the plant and controller states are extracted separately by modifying the least squares identification procedure. It is pointed out that this simplified algorithm leads to a biased estimation of the state sequence for data of finite length. The stability of the identified model can be guaranteed by modifying the extended controllability matrix.

Simulation results from the 4x4 Amoco FCCU, operating in closed-loop with PI controllers, are presented to compare the performance of the Two Step method and the Subspace identification technique. It is indicated that the subspace algorithms identify all the process transfer function quite accurately, whereas the Two Step method identifies only some of the transfer function fairly well.

Chapter 1

Introduction, Motivation and Background

System Identification is quite a diverse and challenging field. A lot of interesting developments have taken place in this area during the last few decades. This has not only improved our understanding about the area but also increased the range of the problems that could be addressed by it. In Section 1.1 we introduce the problem statement of this thesis and explain the motivation behind solving this problem. Section 1.2 explains some basics concepts regarding system identification. The attempt is to introduce this vast area from the user's or practitioners perspective. Section 1.3 outlines the important development which has taken place recently in the area of system identification. It also indicates the future trends and possible direction of research. Section 1.4 explains the organization of this thesis.

1.1 Motivation and Background

A process plant consists of large number of complex unit operations i.e reactors, distillation columns, compressors etc. These unit operations are required to operate according to predefined specifications. A control system causes the process to operate in some desired fashion. In the absence of such a system, the uncertainties in the process behavior (unmeasured disturbances) will drive the process to a different and undesirable operating point. Furthermore, recent trends in economic and environmental conditions have imposed more stringent demands on process productivity, product quality and process flexibility. Higher quality standards are demanded at lower operating costs and with less harmful emissions. The required control performance could be achieved by replacing traditional control strategies (e.g PID) with more advanced control strategies such as Model Predictive Control (MPC). Implementation of MPC requires at least the knowledge of a linear model of the process. Moreover, periodic *modeling of the process* may be required to achieve the desirable state of operation because the process characteristics may change with time.

Process modeling can be classified into several subgroups : the white-box, the black- box and the grey-box modeling. The former method is based on the formalization of fundamental knowledge of a dynamic process, such as physical laws and relations. This approach often leads to a very detailed model for the process. The second approach, black-box modeling , is the development of a model form the observed or collected data and can be referred to as a data driven model.This is also commonly referred as system identification A grey-box model is built from the observed input-output data as well as from the prior knowledge of the process. Typically, a grey-box model is developed from first principles and the value of the involved parameters are evaluated from the observed input-output data. Models identified using the system identification techniques are less detailed and are suitable for the development of a model based controller. In this thesis we just focus on the development of models based on the system identification techniques.

Traditionally, open loop data is used for system identification. This is an undesirable state of operation for plant managers due to safety and product quality requirements. The disturbances affecting the plant may drive the process to an undesirable state if the process is operating in open loop. Hence there is a great need to perform the identification step in closed loop. Focus of the project is to adapt and further develop closed-loop identification methodologies for the identification of black-box models suitable for controller design.

During the early days of feedback control, the controllers were constructed by trail and error. The control theory took a big leap by the introduction of PI and PID controllers and their corresponding tuning rules. (Zeigler and Nichols, 1942) These tuning rules proved successful in most of the industrial application because they provided reasonable initial guesses. Furthermore, the desired performance was obtained by fine tuning the controllers in the field. The drastic increase of energy bills in 1970's lead to the development of energy efficient processes. The developed processes were quite complex in their processing scheme because of the energy integration of the system . Integration of various units lead to the decrease in the energy consumption but only at the cost of more complex and interacting system. The job of control engineer became difficult as the available PI and PID controllers proved inadequate for complex multivariable system. This lead to the development of new branch of control theory called model-based controller design, which expanded the dimension of the problem that could be addressed by effective process control techniques. This explains the growing interest in the area of model-based controller design. The basic assumption made in the application of this technique is that the dynamic characteristics could be described by some model. The black box models are particularly suited for these application because controller design techniques generally require a mathematical model only describing the input-output behavior of the process. (Morari and Zafiriou, 1989; Astrom and Wittenmark, 1989)

A general closed-loop configuration of a multivariable plant is schematically shown in Fig 1.1. The objective is to keep the plant outputs close to their setpoints, even in the presence of unknown disturbances. The typical outputs of the plant which are to be controlled are inventory levels, process temperatures and pressures, product concentration and some of its flowrate. The plant outputs are fed-back to the controller, where they are compared with the desired setpoints. The errors,

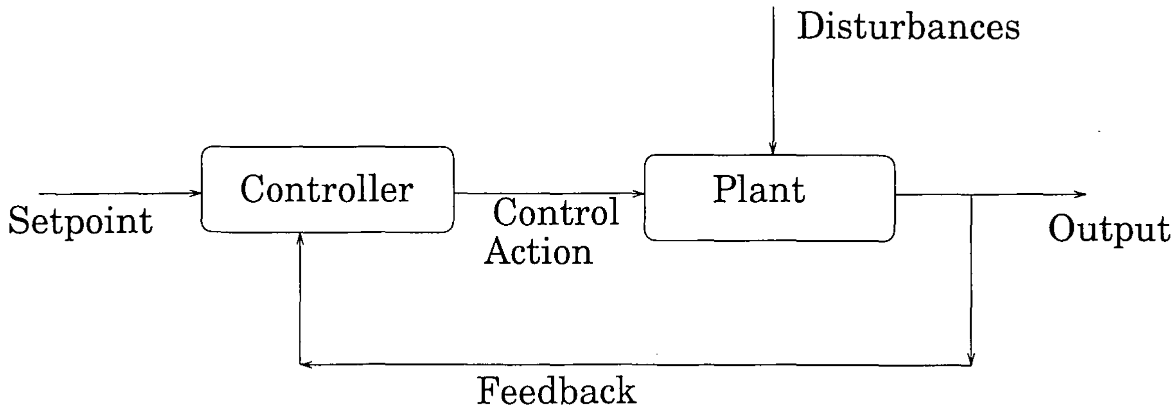


Figure 1.1: A general setup of plant with controller

differences between the plant outputs and the setpoints, is then used to calculate a control signal which drives the process input such that the outputs remain closed to its setpoint. The required knowledge of the system, which the controller uses to calculate the control moves, is obtained from the system identification step i.e from the data set of input-output measurements. The input-output data is usually obtained from experiments which consists of injecting a test signal at the process input and recording the dynamic behavior of the plant outputs as well as the used inputs. The input signal should be designed such that it excites the important plant dynamics. Hence we are ready to state the problem statement of this thesis.

Given a complex multivariable system with unknown dynamics, design the closed loop experiments so as to obtain relevant plant dynamics and develop identification methodologies such that the identified process model could be used in the design of high performance model based controller.

The problem statement basically addresses three major issues:

- Closed Loop Experiment Design
- Development of Closed loop Identification Methodology
- Identification of process model suitable for the design of high performance model based controller

Each of the step is interlinked with the other. The Controller design step will fetch good results only when the model identified form the second step is accurate enough. Moreover, accurate model can be obtained only when the identification experiment is properly designed. But the design of experiments for a given system requires some a priori knowledge about the system dynamics. So the identification task is an iterative procedure. The identified model can be used to determine the frequency range of interest and hence help in the design of the most proper input signal, the second time around.

1.2 System Identification: A Quick Overview

In the area of process control the identification problem can be referred as the process of obtaining a good and reliable process model with a reasonable amount of work. Number of techniques have been developed over the years to accomplice this task successfully. Since these methods are quite general in nature, they leave several choices for the user to tailor them according to his own specific needs. Experimental design, candidate model structure for data representation, estimation method, model validation methods are some of the choices which the user has to make for the application of these techniques.

As indicated in Ljung (1987) and Soderstrom (1989) the experimental design step is one the most important step in identification. The experimental design includes the following aspects about the input and the outputs :

- Which inputs should be excited, how they should be perturbed, how much they should be varied and for how long
- Which outputs should be measured and how often.

The test signal should be selected such that the data obtained is informative enough i.e the input is persistently exciting (Ljung, 1987). The presence of process disturbance and measurement noise mainly affects the variance of the estimated model parameters. The variance of parameters can be reduced by increasing the

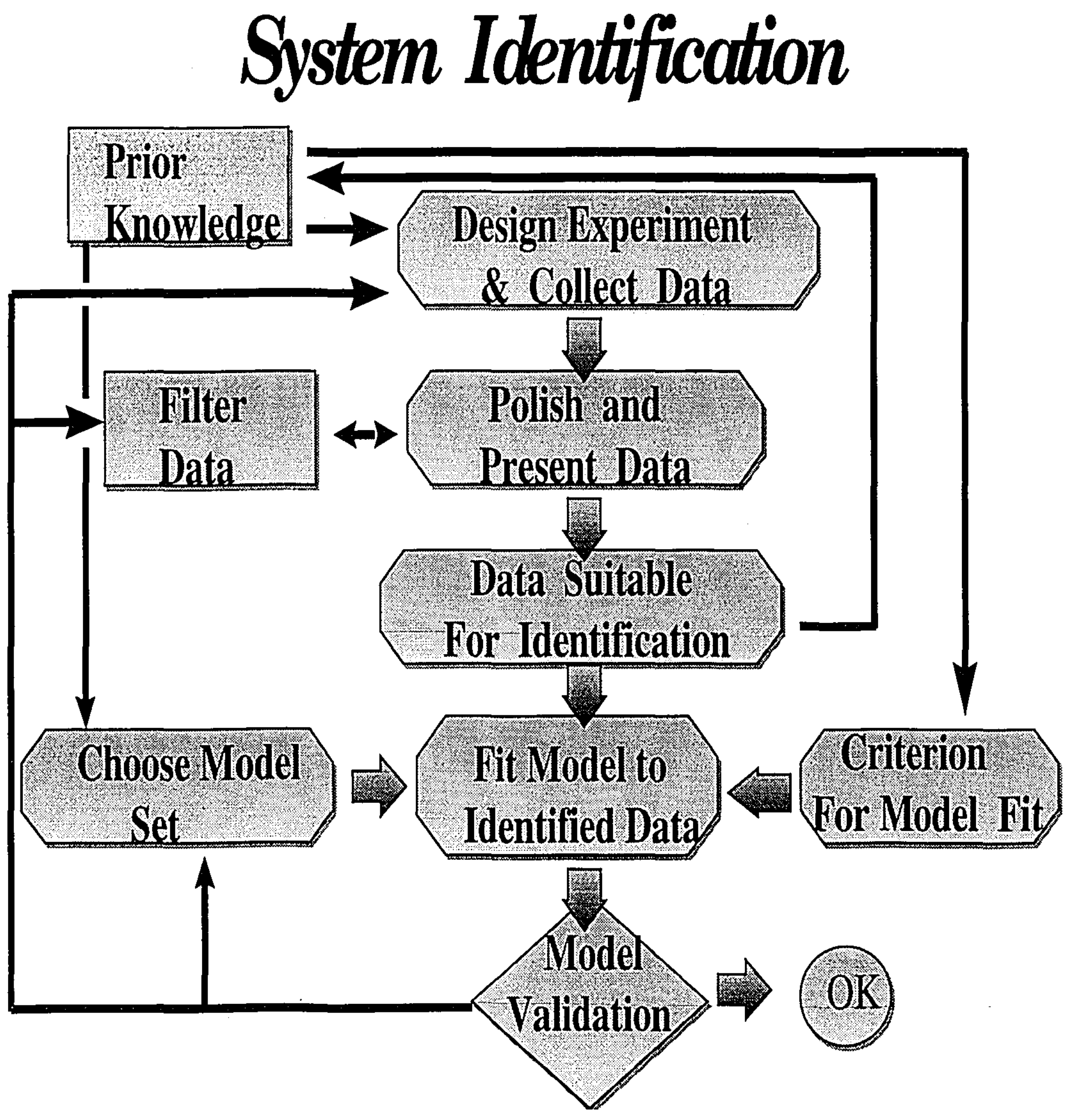


Figure 1.2: A general methodology used for System Identification

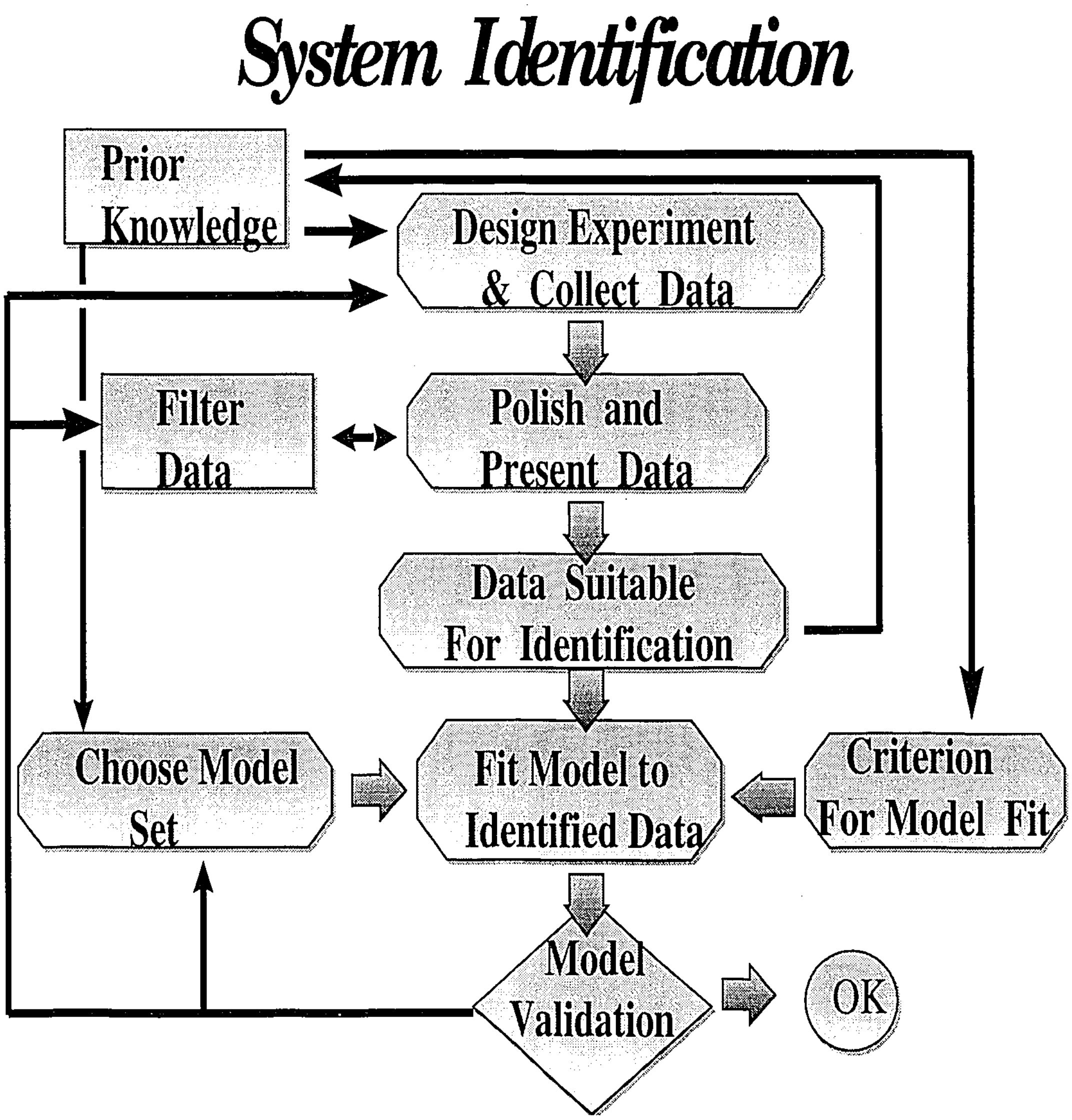


Figure 1.2: A general methodology used for System Identification

length of the identification experiment. In practice the data acquisition is not costless, hence the length of the experiment is dictated by economic considerations. The energy of the test signal should be concentrated in the frequency range where precise information is required about the process. The requirement of informative experiment still leaves the user with a considerable degree of freedom in designing the experiments. Hence one can analyze the "best" experiment within the set of informative ones. This is referred as optimal input design. Survey papers on this aspect details some of the progress made in this area (Goodwin and Payne, 1977; Mehra, 1981; Zarrop, 1979). Depending on the intended model application Gevers and Ljung (1985,1986), Franklin and Kosut (1989) attempted to optimize the experimental design.

Choice of sampling rate is another design aspect during identification. Slower sampling rate leads to data set that is less informative, whereas higher sampling rate leads to poor noise rejection capabilities of the model. Moreover, a model built from data with a sampling interval that is small, compared to the natural time constants of the process, leads to a numerically sensitive procedure (poles are clustered around unit circle). (Ljung, 1987). So a good choice of sampling rate should be a trade off between the noise reduction and the relevance of the process dynamics. The collected data has to be prefiltered before it is used in the identification algorithm. Filtering of data suits two important purpose :

- Removes the information which can cause error in the estimated model
- Improves the model accuracy in the desired frequency range.

Data filtering involves removal of outliers, e.g error in measurement devices which contains no relevant information. Furthermore, least squares prediction error methods are relatively sensitive to this type of error. Low pass filtering of data removes the higher frequency component and avoids the distortion of the frequency spectrum due to aliasing. Removal of mean, linear trends and low frequency drifts, improves the quality of the estimated model.

So far the problem of obtaining the data suitable for dynamic modeling has been discussed. In order to utilize these data for estimation and control design it is necessary to select a model structure to represent the data. Model structure selection basically involves :

- Choice of type of model set. i.e selection between a nonlinear and a linear model, between a parametric or a state space type of model, and so on.
- Choice of size of model in the assumed model set and model parameterization. For parametric models this refers to the choice of degrees of polynomial, whereas for a state space model it refers to the order of state-space model

The quality of the identified model can be viewed in the background of bias and variance. Bias in the identified model can be reduced by increasing the number of parameter in the model structure. But the flexibility obtained is only at the cost of increased parameter variance. So the best model structure is thus a trade off between reducing the bias and the variance of the parameters of the identified model. Given an identified model the quality should be evaluated with respect to the model capability to describe the data, the model complexity and the intended application. Once a model set has been chosen the identification procedure provides us with a particular model in the model set. This model may be the best available one, but the crucial question is whether it is good enough for the intended purpose. Testing if a given model is appropriate is known as model validation. Usually the performance of the the model is evaluated by calculating the error, related to the model accuracy with which it predicts, on a fresh data set and is referred as cross validation. Some important results detailing the effect of design parameters on the bias distribution was reported by Wahlberg and Ljung (1986).

1.3 Recent Development in system identification

Economic and safety reasons have increased the need to perform the identification experiment in closed loop. If the plant is open-loop unstable it is impossible to do an open-loop test without damaging the plant and its environment. Even if the plant is open-loop stable the controller must stay active to react to the disturbances. Otherwise the plant will be producing lot of off-specification product during the open-loop identification tests and hence will result in loss of revenue. Several methods have been presented in the literature to identify a model from closed-loop data i.e direct identification, indirect identification, joint input-output identification. Direct identification consists of straightforward application of prediction error method to the recorded input-output data, neglecting the presence of feedback. The bias introduced by the approximation has been calculated in Ljung (1987). The identifiability conditions have been derived by Anderson and Gevers (1982), Wellstead (1978) and Ljung *et. el* (1974). .

Indirect identification consists of two sequential steps. First, the closed loop transfer function between external signal and the plant output is identified. The basic assumption made during the application of indirect identification is that the controller transfer function is known. Estimates of the closed loop transfer function and the knowledge of the controller is then used to calculated the plant transfer function. Properties of indirect identification have been considered by Ljung *et. el* (1974), Soderstrom*et. el* (1975), Gevers (1978) and Zheng and Feng (1991). In the joint input-output method the closed loop system is viewed as black box with white noise source as input and the process input and output as the joint output. The resulting black box model can be estimated using prediction error method (Soderstrom and Stoica, 1989), spectral factorization (Anderson and Gevers, 1979) or stochastic realization (Van der Klauw and den Bosch P.P.J, 1991). The identifiability with joint input-output has been considered by Sin and Goodwin (1980), and by Aling and Bosgra (1990).

Application of most of the methods discussed above, results in the estimation of biased model. It has been realized that due to undermodeling there will always be a plant model mismatch. This has lead to the development of the algorithm Two Step Method which controls the bias distribution explicitly. The Two Step Method of Van den Hof and Schrama (1992,1993) , controls the bias of the identified model by solving the identification problem in two steps, each of which uses the open loop identification technique. Control of bias distribution during identification has added a new dimension in the area of identification. In control literature it is commonly known as control-relevant identification. The idea is based on the fact that the model has to be more accurate in certain frequency range for the design of high performance controller. This design aspect is taken care during identification, to come up with models which have lower uncertainty in the desired frequency range. Several algorithms incorporating the identification step in the controller design step has been proposed in the literature. (Schrama, 1992; Shook et al., 1992; Astrom, 1993; Rivera and Bhatnagar, 1992; Rivera et al., 1992).

Traditional system identification techniques like the Prediction Error Method (PEM) (Ljung, 1987) and the Instrumental Variable Method (IV) (Soderstrom and Stoica, 1989), performs well on many systems. But they fall apart as the complexity of the system increases. Use of PEM for multivariable cases results in a nonlinear optimization problem with large number of unknowns (> 100), for which the optimal parameter estimation is practically impossible. The solution to this problem is the use of Orthonormal basis functions (Heuberger, 1991) so that the optimization problem reduces to a least squares one. The development of a state-space model is another important approach. Subspace based state-space identification methods result in a numerically reliable state-space model even for complex multi-variable dynamical systems.

Most of the a priori parameterization can be avoided by using the state space approach. The only input required is the number of block rows to be used for defining the Hankel matrix (Van Overschee, 1995). The order of the system is determined through the inspection of dominant singular value of the oblique projection matrix which is calculated during identification. Subspace identification algorithms are the input-state-output generalization of the classical realization theory as developed in the sixties. Classical Identification scheme (Ho and Kalman, 1966) identifies a state-space model from the impulse response (Markov Parameters). Number of algorithms has been proposed to improve the robustness of this algorithm. (Dicksinson et al., 1974b; Dicksinson et al., 1974a; Kung, 1978; Zeiger and McEwen, 1974). Most of them introduce the SVD as a tool to reduce the sensitivity to errors in the measured impulse response. Furthermore, recently number of algorithm extending this idea has been reported in the literature. (Ljung, 1991; King et al., 1988; Liu and Skelton, 1991; Bayard, 1992). A non-parametric approach for the estimation of impulse response is the use of frequency domain identification. Mackelvey uses the inverse discrete Fourier transform for the non-parametric estimation of the transfer function (McKelvey et al., 1994; McKelvey, 1995b; McKelvey, 1995a). This approach is interesting from the viewpoint of control-relevant identification because the bias in the frequency domain can be directly affected during the identification. The class of techniques, which identify the impulse response coefficients from the input-output data, is referred as realization-based subspace identification methods.

Realization-based subspace algorithms requires special input sequences such as impulse response or white noise sequences, hence are difficult to apply in practice. This lead to the development of another class of identification algorithm which are referred as direct subspace identification algorithms. The alternate approach is the estimation of the observability and/or the controllability matrices without forming the impulse response coefficients (Moonen et al., 1989; Moonen and Vanderwalle, 1990; De Moor et al., 1988; Verhaegen and Deprettere, 1991; Verhaegen, 1993; Verhaegen, 1994; Viberg M. and Ljung, 1993). Meanwhile, the stochastic realization problem can also be solved to get the noise characteristic. The pioneering work of Akaike (Akaike, 1974; Akaike, 1975) introduced the canonical correlations in the stochastic realization framework. Useful insights can be obtained form recent work on stochastic identification problem. (Arun and Kung, 1990; Desai and Pal, 1984; Desai et al., 1985). Several techniques has been proposed recently to solve the combined (deterministic-stochastic) identification problem (Larimore, 1990; Verhaegen, 1991; Verhaegen and Dewilde, 1992; Van Overschee and De Moor, 1994b).

Some good review papers have been recently published on this subject which describe the whole class of subspace algorithms (Rao and Arun, 1992; VanDer Veen et al., 1993; Viberg, 1994). The area of subspace-based system identification is still not very mature. The problem which is yet to be answered is the accuracy of the estimation from finite length of data. For PEM, the expression for the parameter variance and the approximation performance (the bias distribution) is well-known (Ljung, 1987). Whereas for subspace identification the results concerning the asymptotic bias in the case of undermodeling is yet to be published. An effort of relating subspace identification to the classical techniques is suggested in Janson (1994), where the unknown state sequence is replaced by a reconstructed state sequence from an optimal observer. (Jansson and Wahlberg, 1994). An interesting interpretation connecting the weighting of oblique projection in subspace identification with the pre-filtering in PEM is presented by Van Overschee. (Van Overschee and De Moor, 1993b; Van Overschee and De Moor, 1994c; Van Overschee and De Moor, 1994d; Van Overschee and De Moor, 1994a)

1.4 Organization of the Thesis

Chapter 1

Explains the motivation behind the closed loop system identification and states the problem addressed in this thesis. It gives a quick overview about various aspects of system identification. Moreover, the traditional techniques and some recent development are discussed to get some useful insight about the closed loop identification problem and indicates the trends in the future research.

Chapter 2

Classical identification schemes like Prediction Error Method (PEM) and Instrument variable (IV) Method are discussed in detail. The advantages and disadvantages of traditional closed loop identification techniques like direct identification, indirect identification and joint input-output identification are discussed. The frequency domain expressions for each of the methods are analyzed to get some knowledge about the possible bias distribution. Furthermore, a known SISO system is considered to provide some guideline about the possible injection point of the external signal in the closed loop. A recent development in the area of closed identification, the Two Step Method, is also described in detail. The identification results for the Amoco FCCU using the Two Step method and the direct identification are compared to prove the effectiveness of Two Step method.

Chapter 3

This chapter introduces the subspace based state space identification algorithms. The geometric tools regarding matrix projections is discussed. Since matrix projection forms the basis for subspace identification algorithm, concept like the vector space, column space, row space etc. are dealt in detail. The chapter introduces the simple problem of subspace identification of deterministic systems, where process noise and measurement noise are assumed to be zero. Even though the results have been published somewhere else in this area, we discuss it here because we want to introduce the reader to the area of subspace system identification. Moreover, the open-loop deterministic technique forms the basis for the closed loop problem which is discussed latter in the thesis. The combined deterministic-stochastic algorithm is also discussed to give a complete picture about the identification problem. In each method, it is discussed how the system states can be recovered from the plant input-output data. The reader should refer to the following publication for fine details and more through treatment of the subject. (De Moor, 1988; Van Overschee and De Moor, 1992; Van Overschee and De Moor, 1993a; Van Overschee and De Moor, 1994b; Van Overschee and De Moor, 1994c; Van Overschee and De Moor, 1994d; Van Overschee, 1995).

Chapter 4

Open loop identification techniques are not directly applicable to closed loop data due to correlation between input and unmeasured disturbances. Current subspace theories and methodologies support state-space identification for open-loop data only. Hence there is a need to develop a technique such that it could be applied to closed loop data. The chapter describes such a methodology which extends the open-loop identification algorithm to closed-loop data by incorporating a modification in the open-loop identification algorithms. The validity of the method is proved using the linear algebra concepts. The chapter summarizes the derivations which were done to support the development. In addition a technique to increase the robustness of the algorithm is also proposed. The technique guarantees the stability of the identified model whereas traditional methods tends to compute unstable systems for lightly damped poles. The stability aspect is really critical when identification is done online, so the proposed identification scheme is suitable for online application too. Simulation results from a 4th order SISO system indicates that the proposed algorithm does a reasonable job of capturing the system dynamics. Moreover, the result suggests that controller transfer function can also be identified accurately. The suggested stability modification was applied to a 2nd order lightly damped system and the validity of the technique was proved.

Chapter 5

The chapter presents the application of the developed closed loop subspace algorithms on a MIMO system. The algorithm was applied to a 4x4 Amoco FCCU operating in closed loop and the results are compared with the Two Step algorithm.

Chapter 6

Summarizes the present work on system identification. The research in this area is far from over as lots of question of practical importance are yet to be answered. The major issues in this area was spotted and was listed as future work. Possible ways of improving the current theory has also been indicated.

Chapter 2

System Identification : Prediction Error Method

In this chapter we present the basic tools required for process identification. The focus here, is to discuss the classical and the recent development in the area of Prediction Error Method. In Sec. 2.2 we discuss the open loop identification methods. Sec 2.3 introduces the techniques which have been developed in the area of closed loop identification. Sec. 2.3.4 discusses one of the promising closed loop identification methodology namely Two Step Method. Frequency domain expression for Two Step Method is analyzed to evaluate the bias distribution for the case of undermodeling. Simulation example for open and closed-loop identification are presented in Sec 2.4 to get some some insight into the theory which is discussed in this chapter. Some of the closed loop identification issues like, location of external signal in the loop, design of input signal, is treated in Sec 2.4. Since most of the real life measurements are corrupted with noise, results of noise to signal ratio on the identification methodology is discussed in Sec 2.4.2

2.1 Preliminaries

We will assume that the true process can be described as

$$S_T : y_k = G(z)u_k + H(z)e_k \tag{2.1}$$

where y_k is the process output, u_k is the process input and e_k is a zero mean random signal. $G(z)$ and $H(z)$ are process and disturbance transfer functions respectively. We wish to estimate the model for the plant of the form

$$M : \hat{y}_k = \hat{G}(z,\theta)u_k + \hat{H}(z,\theta)e'_k \tag{2.2}$$

where e'_k represents the prediction error for the model with $\hat{G}(z)$ and $\hat{H}(z)$ as process and disturbance transfer function respectively. Our goal is to perform estimation of the parameter vector θ such that the model captures the important dynamic properties of the plant. To extract enough information about the process, the input signal need to be sufficiently rich or *persistently exciting* (Ljung 1987).

2.2 Open-Loop identification

The Literature is quite rich in the area of open loop identification. Important inferences can be made about the closed loop identification problem from the open loop techniques. Hence the open-loop techniques should be addressed before we delve into the close-loop problem. Several approaches can be taken to tackle the open loop problem. They can be broadly classified as follows :

2.2.1 Parametric Identification

Prediction Error Method

Given the description Eq. 2.2 and having at hand the input-output data (u,y), the one step ahead prediction error can be computed as (see Ljung(87) for the basics theory)

$$e_k(\theta) = y_k - \hat{y}_{k|k-1}(\theta) = L(z)\hat{H}(z,\theta)^{-1}\left[(G(z) - \hat{G}(z,\theta))u_k + H(z)e_k\right] \tag{2.3}$$

$$\text{where } (\hat{G}_N(z,\theta), \hat{H}_N(z,\theta)) = argmin\frac{1}{N}\sum_{K=1}^{K=N} L(z)\left[e_k(\theta)\right]^2 \tag{2.4}$$

The objective function for parameter estimation is to minimize the squared filtered prediction error. Prediction errors are prefiltered, by $L(z)$, to facilitate better control over the frequency domain bias in the estimated process model. Eq 2.2 can be written equivalently in the frequency domain as

$$\lim_{N\to\infty} V = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(|G - \hat{G}(\theta)|^2\Phi_u(\omega) + \Phi_v(\omega)\right)|L\hat{H}^{-1}(\theta)|^2 d\omega \tag{2.5}$$

representing the output error spectrum as

$$\Phi_{ERR} = \left(|G - \hat{G}(\theta)|^2\Phi_u(\omega) + \Phi_v\right) \tag{2.6}$$

where Φ_u and Φ_v represent the power spectra for the input and disturbance variables, respectively. Eq. 2.5 illustrates the fact that $\hat{G}(\theta)$ converges to the model, within the assumed model set, that minimizes a frequency weighted integral of the squared error with a frequency weighting, a function of input signal power spectral density Φ_u, prefilter transfer function $L(z)$ and the noise model $\hat{H}(\theta)$. One important advantage of open loop identification is that consistent estimates of the process transfer function can be obtained, even if the assumed noise model is incorrect, as long as $\hat{H}(\theta)$ and $\hat{G}(\theta)$ are independently parameterized. This is because of the fact that model parameters $\hat{G}(z,\theta$ and the unknown noise characteristic Φ_v appear separately in the optimization criteria (Eq. 2.5).

The optimization criteria given by Eq.2.5 will result in an accurate plant model for the frequency range where input spectrum is higher in magnitude. In other words, the bias $|G - \hat{G}|$ will be small where Φ_u is higher in magnitude. For the case of independently parameterized noise model the parameter of the noise model converges to the value such that the model noise spectrum $|\hat{H}(z,\theta)|^2$ resembles the error spectrum Φ_{ERR} as much as possible, within the chosen model noise spectrum.

Ideally Φ_{ERR} should be white noise and hence $\hat{H}(z,\theta)^{-1}$ should be the inverse of the actual noise transfer function. For the case when the plant and disturbance model have some common parameters, no clear cut estimate of the resulting model accuracy could be given. Intuitively speaking, the common parameters in the process and the disturbance model will fit the process model in the higher frequency range. Hence an ARX model will result in high frequency fit. To avoid this problem the plant model and the noise model should be independently parameterized and the possible choice of models for this case are Output Error (OE) and Finite Impulse Response (FIR) model. ,The main features of the model obtained form the open loop identification are:

1. In the case of $S_T \in M$, the objective function is minimal when $G(z) = \hat{G}(z,\theta)$ and $H(z) = \hat{H}(z,\theta)$ and we get consistent estimates of both the process and disturbance transfer function.

2. For the case of $S_T \notin M$ and $G \in M$, we can get consistent estimate of G irrespective of $\hat{H}(z,\theta)$ by using independent parameterization of $\hat{G}$ and $\hat{H}$. This is an important issue form the controller point of view since we are more interested in obtaining an accurate estimate of $G(z)$ and not the noise/disturbance transfer function.

3. Often the case is that the models $\hat{G}(z,\theta)$ is only a lower order approximation of the actual process and we may have a situation in which $P \notin M$. In this case, we can still influence the bias distribution by proper choice of the input spectrum and/ or the prefilter $L(z)$.

These are important characteristics of an identification scheme since they establish the conditions under which consistent estimate can be obtained.

Instrumental Variable Methods

The predictor model can be expressed in a linear or pseudo-linear regression model. Regressors are constructed from the past input and output data. Least square estimate of θ can be expressed as

$$\hat{\theta}_N^{LS} = sol\left\{\frac{1}{N}\sum_{t=1}^{N}\Phi(t)\left[y(t) - \Phi^T(t)\theta\right] = 0\right\} \tag{2.7}$$

Due to correlation between $v_0(t)$ and $\phi(t)$, θ_N will not tend to θ_0 in typical cases. So general correlation vector $\xi(t)$, instrumental variables, can be used which has the property $\overline{E}(\xi(t)v_0(t)) = 0$. This gives

$$\hat{\theta}_N^{IV} = sol\left\{\frac{1}{N}\sum_{t=1}^{N}\xi(t)\left[y(t) - \Phi^T(t)\theta\right] = 0\right\} \tag{2.8}$$

The models presented in this thesis are estimated primarily using Prediction Error Method (PEM). A good initial guess of parameters are required particularly for MIMO case because of local minima problems. In these cases instrumental variables could serve as an handy tool to form an initial estimate for the PEM approach.

2.2.2 Non-parametric Identification

A non-parametric method provides the model for $G(z)$ only. No parameterization is done in this case. Since only a small data reduction is achieved, the variance of the estimate is quite high. Hence this technique should be applied only when sufficient measurements are available, otherwise it could lead to highly misleading results. Techniques, like windowing, are applied to smoothen (can also be referred as variance reduction) the estimate but only at the cost of the introduction of bias. If the window is "wide", then many frequencies will be weighted together in Eq. 2.11. This will lead to a smaller variance in the transfer function estimation. But at the same time it will result in increased bias. So the width of the window will thus control the trade-off between bias and variance. Some common windows used for this purpose are Bartlett, Parzen and Hamming. Non-parametric method serves as a good validation tool for parametric identification. Equation 2.1 can be interpreted in frequency domain as

$$\Phi_y(\omega) = |G(e^{i\omega})|^2 \Phi_u(\omega) + \lambda |H(e^{i\omega})|^2 \tag{2.9}$$

$$\Phi_{yu}(\omega) = |G(e^{i\omega})|^2 \Phi_u \tag{2.10}$$

the spectral estimate is obtained from the input output data by applying suitable lag window W_M. The estimates are then formed as (Ljung 1987)

$$\hat{\Phi}_u(\omega) = \sum_{\tau=-M}^{\tau=M} \hat{R}_u(\tau) W_M(\tau) e^{i\omega\tau} \tag{2.11}$$

$$\hat{G}_N(e^{i\omega}) = \frac{\hat{\Phi}_{yu}(\omega)}{\hat{\Phi}_u(\omega)}; \quad \hat{\Phi}_v(\omega) = \hat{\Phi}_y(\omega) - \frac{|\hat{\Phi}_{yu}(\omega)|^2}{\hat{\Phi}_u(\omega)} \tag{2.12}$$

2.3 Closed Loop Identification

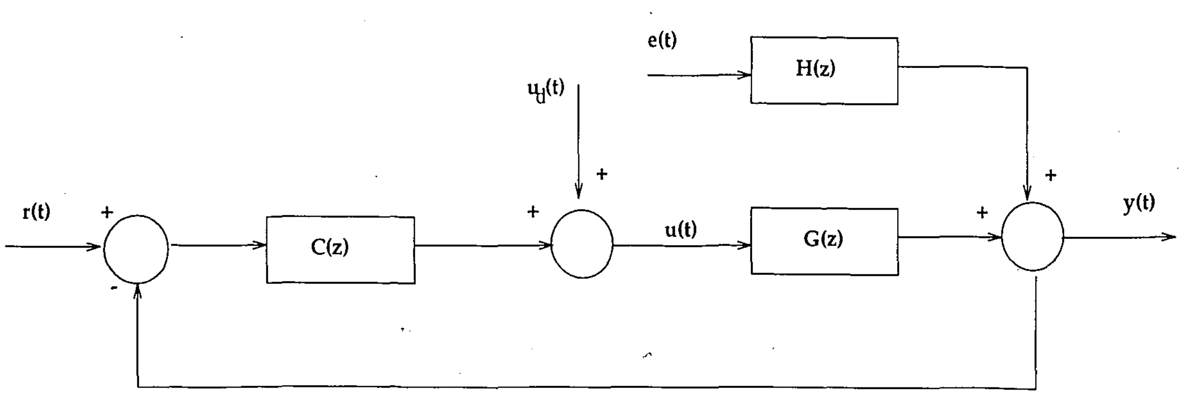


Figure 2.1: Closed loop system configuration

It is assumed that under the influence of feedback controller $C(z)$ the closed loop system is stable. The closed loop identification problem is to estimate process model $\hat{G}(\theta)$ from the closed loop data $\{y_N, u_N\}$ as shown in figure 2.1. It is assumed that the external signal $u_d(t)$ or $r(t)$ is present. The relationships between various signals can be given by equation 2.14 and 2.13, whereas $S(z) = (1 + P(z)C(z))^{-1}$ is the closed loop sensitivity function.

$$u(t) = S(z)u_d(t) + C(z)S(z)r(t) - S(z)C(z)H(z)e(t) \quad (2.13)$$

$$y(t) = S(z)P(z)u_d(t) + P(z)S(z)r(t) + S(z)H(z)e(t) \quad (2.14)$$

Given the closed loop configuration of Fig 2.1, we are interested in a solution of the closed-loop identification method such that the attractive features of the open-loop identification problem are retained. The open-loop prediction error identification methods applied to closed loop data, can yield parametric model that describe that dynamic behavior of the process. However, the identification criterion as given by Eq. 2.5 and the subsequent interpretation is not valid when applied to the closed loop data due to the correlation between u and e and the inherent difficulties associated with the closed loop identification. The problems in closed loop identification are due to identifiability and bias.

Identifiability: It refers to the ability to estimate the model parameters using the closed-loop data. There are two types of identifiability concepts given in literature (Rivera and Bhat ,92)

- *System Identifiability (SI):* Model parameter estimates that correctly represent the dynamic behavior of the process can be obtained from the data provided that certain conditions are met with regards to the model parameterization, controller configuration and experimental conditions.

- *Strong System Identifiability (SSI):* This property means that no special restrictions are imposed on the model parameterization or the controller to be able to obtain parameter estimates.

Bias : Bias refers to the error between the model and the process transfer function that occur in the identified model due to factors as model parameterization, input signal and the experimental conditions. We have seen the effect of bias while discussing the open-loop identification. These errors persist even with infinite number of measurements are available and the identifiability conditions are satisfied.

2.3.1 Direct Identification :(DI)

Direct identification consist of straightforward application of a PEM to closed loop data ignoring the effect of feedback. The frequency domain objective function for this case is give as

$$\lim_{N\to\infty} V = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(|G-\hat{G}(\theta)|^2\left[\Phi_{ud}(\omega)+|C|^2\Phi_r(\omega)\right]\right)|SL\hat{H}^{-1}(\theta)|^2 d\omega$$
$$+\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(|1+\hat{G}(\theta)C|\Phi_v(\omega)\right)|SL\hat{H}^{-1}(\theta)|^2 d\omega \quad (2.15)$$

It is not possible to control the bias distribution in the estimation of $\hat{G}(\theta)$ because it is affected by the unknown disturbance characteristic, by the presence of second term in Eq. 2.15, even if the process and the disturbance transfer function is independently parameterized. In other words, the model parameters $\hat{G}(z,\theta)$ are now directly effected by the unknown noise characteristic, Φ_v. This is a serious drawback of the direct identification and leads to a recent development in closed-loop identification,referred as the Two Step method, which is described later in this section. In spite of the serious drawback DI is still an attractive method due to its simplicity. In the absence of any external signal the model will try to approximate the inverse of the controller. Key features of the identification results based on the Eq. 2.15 are

1. If $S_T \in M$, the objective function is minimal when $G(z) = \hat{G}(z,\theta)$ and $H(z) = \hat{H}(z,\theta)$ and we get consistent estimates of both the process and disturbance transfer function.

2. However, if $S_T \notin M$ the estimate of $P(z)$ would be biased even if $P \in M$. Thus is to say that we can not obtain unbiased estimates of the process transfer function even with a correct parameterization if the disturbance transfer function is not modeled correctly.

3. The model fit is influenced by the controller in two ways. First the controller is present in the closed loop transfer function which weigh the bias term $P-\hat{P}$. We can also see the effect of the controller in $(1+\hat{P(\theta)})^{-1}$ which weighs the disturbance transfer function.

4. External signal is needed for closed-loop identification. Without the presence of either $u_d(t)$ or $r(t)$, the model will try to approximate the inverse of the controller, $-C^{-1}$.

5. Also, we do not have direct control over the bias in the estimation of $\hat{G}(z,\theta)$ in the case of under modeling since the bias distribution still depends on the unknown distribution characteristic due to the second term in Eq.2.15. In this case, even the independent parameterization of process and the disturbance transfer function does not eliminate the bias unlike the case of open loop identification problem.

From the practical point of view the identifiability conditions can be assured by injecting a persistently exciting signal.

2.3.2 Indirect Identification : (II)

Indirect identification Method consists of two sequential steps:

1. The closed loop transfer function is estimated between the external input and the output.

2. Estimate of the process transfer function is obtained from the known controller transfer function and the estimate obtained from the first step.

For the case when the signal is injected at the setpoint, the first step involves the identification of $\hat{G}_c(z) = G(z)S(z)C(z)$ and $\hat{H}_c = S(z)H(z)$ as process and disturbance transfer function respectively. The model $\left(\hat{G}(z,\theta), \hat{H}(z,\theta)\right)$ is obtained as

$$\hat{G}(\theta_N) = \left[Ip - \hat{G}_c(\theta_N)C_b\right]^{-1}\hat{G}_c(\theta_N); \quad \hat{H}(\theta_N) = \left[Ip - \hat{G}_c(\theta_N)C_b\right]^{-1}\hat{H}_c(\theta_N) \tag{2.16}$$

whereas $\left(\hat{G}_c(\theta_N), \hat{H}_c(\theta_N)\right)$ is the closed loop model obtained from $\{y_t, r_t\}$ in the first step. $\hat{G}(\theta_N)$ can be estimated correctly only if the model in the first step is determined accurately, which is impractical as the unmeasured disturbances effect the bias distribution. Similar to direct identification the identifiability conditions are satisfied in the presence of an external signal. But the assumption the the controller transfer function is known is quite restrictive for the application of this technique. It may not be always possible to obtain this information in practice.

2.3.3 Joint Input-output Identification : (JIO)

In Joint input-output identification the closed loop system is treated as a black box with white noise as an input and the process input and output variables as the output variable of the closed-loop black-box system. The closed loop system can be described as

$$Z_t = \begin{pmatrix} y_y \\ u_t \end{pmatrix} = \begin{bmatrix} W_{11} W_{12} \\ W_{21} W_{22} \end{bmatrix} \begin{pmatrix} \xi_{y,t} \\ \xi_{u,t} \end{pmatrix} \tag{2.17}$$

where $\xi_{y,t}$ and $\xi_{u,t}$ are the uncorrelated white noise input to the system. It can be shown that $\hat{G}(\theta_N) = \hat{W}_{12}(z)\hat{W}_{22}(z)^{-1}$. Either of the three method, PEM, spectral factorization or stochastic realization could be applied to estimate W.

2.3.4 Two Step Method

Two step (Van den Hof and Schrama 1993) method is a recent development in the area of closed loop identification. Consistent estimate of plant transfer function can be obtained from the closed loop data even in the situation where the noise model is not accurate. Moreover, explicit expression for the bias distribution can be obtained of the resulting model. Steps involved in the identification methodologies can be summarized as

1. The closed loop sensitivity function is identified from the closed loop data using high order linear model. The identified model is used to reconstruct the noise free input signal.

2. The reconstructed signal then is used to identify the process transfer function.

$$u(t) = S_0(q)\left[u_d(t) + C(q)r(t)\right] - S_0(q)C(q)H_0(q)e(t) \tag{2.18}$$

$$y(t) = S_0(q)G_0(q)\left[u_d(t) + C(q)r(t)\right] + S_0(q)H_0(q)e(t) \tag{2.19}$$

whereas $u^n = S_0\left[u_d(t) + C(q)r(t)\right]$ is the reconstructed noise free input, hence the process transfer function can be estimated by

$$y(t) = G_0(q)u^n + S_0(q)H_0(q)e(t) \tag{2.20}$$

This method reduces the closed loop problem to two open loop identification problems. In both the steps one step ahead prediction error criteria is used for identification. The open loop identification criteria can be used in both the steps because the inputs u^d, u^r (first step) and u^n (second step)are uncorrelated with the noise $e(t)$. Identification criterion in f.frequency domain can be represented as (Van den Hof and Schrama 1993)

Frequency Domain Expression for First Step :

$$\begin{aligned}\lim_{N\to\infty} V(\beta) = &\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(|S_0 - \hat{S}(\beta)|^2\Phi_{u^n}(\omega)\right)|L_1\hat{H}_u^{-1}(\beta)|^2 d\omega \\ &+\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\Phi_v(\omega)C(\omega)S_0(\omega)\right)|L_1\hat{H}_u^{-1}(\beta)|^2 d\omega\end{aligned} \tag{2.21}$$

Frequency Domain Expression for Second Step :

$$\begin{aligned}\lim_{N\to\infty} V(\theta) = &\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(|G_0S_0 - \hat{G}(\theta)\hat{S}(\beta)|^2\Phi_{u^n}(\omega)\right)|L_2\hat{H}_y^{-1}(\theta)|^2 d\omega \\ &+\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\Phi_v(\omega)S_0(\omega)\right)|L_2\hat{H}_y^{-1}(\theta)|^2 d\omega\end{aligned} \tag{2.22}$$

The first and second step are parameterized with parameter vector β and θ respectively. Where as $H_u(\beta)$, $H_y(\theta)$ are noise model, and L_1, L_2 are the prefilter. Since noise model and the process model should be independently parameterized, high order FIR, OE or Orthogonal FIR can be used for this purpose. The first part in the Eq. 2.22 can be represented as (Van den Hof and Schrama 1993)

$$\left[G_0S_0 - G(\theta)S(\beta)\right] = \underbrace{\left[G_0 - G(\theta)\right]S_0}_{Secondstep} + \underbrace{\left[S_0 - S(\beta)\right]G_\theta}_{firststep} \tag{2.23}$$

the result shows that the error in estimation of $G(\theta)$ consists of part which comes from the first and the second step. It shows that the consistent estimation of sensitivity function S_0 is not mandatory to get good approximation of process transfer function. The requirement of small error in the sensitivity function is sufficient to model the input- output transfer function accurately. Comparing Eq. 2.21,2.22 with 2.15 it is evident that both two step and direct identification will yield the same result if no disturbance is present or the applied input spectrum has larger dominance, over that of the noise signal, in the frequency range of interest. Hence the first step can be thought of as a noise rejection step. It removes the correlation between the input and the unmeasured disturbance, so the consistency property of open loop identification can be invoked. If the reconstructed input signal is such that it resembles the original noisy input signal then not much can be expected from thé two step method because then it reduces to the DI approach.

2.3.5 Non Parametric Identification

Soderstrom and Stoica (1989) have shown that nonparametric identification does not give satisfactory results for closed loop data. The spectral estimate tries to compromise between the real process and the inverse of the controller, $-1/C_b$. This problem can be avoided by using an external input. Asymptotic unbiased estimate can be obtained, provided the spectra $\hat{\Phi}_{yr}(\omega)$ and $\hat{\Phi}_{ur}(\omega)$ can be estimated consistently.

$$\hat{\Phi}_{yr}(\omega) = \hat{G}_0(e^{i\omega})\hat{S}_0(e^{i\omega})\hat{\Phi}_r(\omega); \quad \hat{\Phi}_{ur}(\omega) = S_0(e^{i\omega})\hat{\Phi}_r(\omega); \tag{2.24}$$

$$\hat{G}_N(e^{i\omega}) = \hat{\Phi}_{yr,N}(\omega)\hat{\Phi}^{-1}_{ur,N}(\omega) \tag{2.25}$$

2.4 Simulation Example: FCCU Simulation

2.4.1 Open Loop Identification:

The rigorous model for Amoco FCCU was developed by combined efforts of CPMC at Lehigh University and Amoco corporation (McFarlane et al. 1993). The details about the process in given in Sec 5.2.1. This dynamic model captures the important nonlinearities, multi-variable interaction along with the constraints. The controlled variables are: 1)Reactor riser temperature, T_r 2) Regenerator Bed temperature, T_{reg} 3) Stack gas oxygen concentration, $O2_{sg}$ 4) Stack gas carbon monoxide concentration, CO_{sg} and 5) Wet gas compression suction valve position, V_{11} . The manipulated variables are 1) Feed flow rate, $F3_{set}$ 2) Slurry recycle flow rate, $F4_{set}$ 3) Liftair flow rate, $F9_{set}$ 4) Reactor-Regenerator differential pressure, D_{pset}.

Sampling Rate: Sampling of system leads to information losses. So it is important to select the sampling interval so that the losses are insignificant. It is well known that the information for the frequency higher than ω_N *(Nyquist frequency)* is lost due to the aliasing effect. For costless data aquisition higher sampling rate seams to be the first choice but this leads to problems like poor noise rejection, model fit in the higher frequency region and numerically sensitive procedures (Ljung 1987). From the studies by (Kalra and Georgakis 1995) it was found that largest ultimate frequency ω_u was around 0.0139 rad/sec. So the sampling time of 5 sec seams a safe choice. The data can always be resampled to increase the low frequency fit.

Excitation Signal (SISO case): Pseudo random binary signal (PRBS) (McFarlane and Rivera, 1992), and Generalized Binary Signal (GBN) (Tulken, 1990) are the common sequences used for identification purpose. Only PRBS design has been considered for this report. T_{cl} (minimum switching time) and N (No. of shift registers.) are the two important design parameters for PRBS design. To cover the desired frequency range a combination of two PRBS signal was chosen. A switching time of 100 sec with 5 shift registers was found to be a good choice for high frequency content, whereas switching time of 250 sec and 6 shift register covered the intermediate frequency range. The minimum length of PRBS test for the former case can be calculated as $(2^n - 1)T_{cl} = 3100sec$, whereas for the latter case it is 15750 sec. The test was conducted for the duration of two input sequences. Hence the combined input (both PRBS signals) was conducted for the length 40,000 sec. A total of 8000 samples were generated for the sampling rate of 5 sec. Amplitude of excitation was chosen by trial and error for each of the input.

Identification Results : Generated data was treated to remove the linear trends. It was also checked for outliers by visual inspection. Different model structure, Output Error (OE) and Auto Regressive eXtra (ARX), [1] [2] was considered for identification purpose. For a given model structure AIC (Akaike's Information Criterion) or FPE (Final Prediction Error) criterion can be applied to come up with suitable model order. Its difficult to apply the AIC or the FPE criteria for the MISO case because of the number of iteration involved. Since MISO results are to be compared with SISO, similar validation techniques were used for both the cases. A model which had acceptable time domain fit (represent high frequency content), suitable step response (low frequency content) and good residual analysis was chosen as the final model. High or low frequency fit can be influenced by choosing suitable model structure. Cross validation was done on the data set having twice the amplitude of the identification data. As seen in the Fig. 2.2 and 2.3 the OE model does a good job of capturing the low frequency as well as high frequency information. Since the process is quite nonlinear for Cosg their is a difference in the actual steady state and the predicted steady state values.

2.4.2 Closed Loop Identification:

Excitation Signal Issues: As mentioned in the introduction to the closed-loop identification problem, an external signal can be injected at two different places in the loop. Once an excitation signal with the desired properties has been designed, the proper location, $u_d(t)$ or $r(t)$, for the signal injection should be chosen.

[1]The Output Error model is given as

$$y(t) = \frac{b_1 + b_2 q^{-1} + b_3 q^{-2} + \cdots + b_{n_b} q^{-n_b+1}}{1 + f_1 q^{-1} + f_2 q^{-2} + \cdots + f_{n_f} q^{-n_f}} u(t - n_k) + e(t)$$

[2]The ARX model is given as

$$\left[1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}\right] y(t) = \left[b_1 + b_2 q^{-1} + \cdots + b_{n_b} q^{-n_b+1}\right] u(t - n_k) + e(t)$$

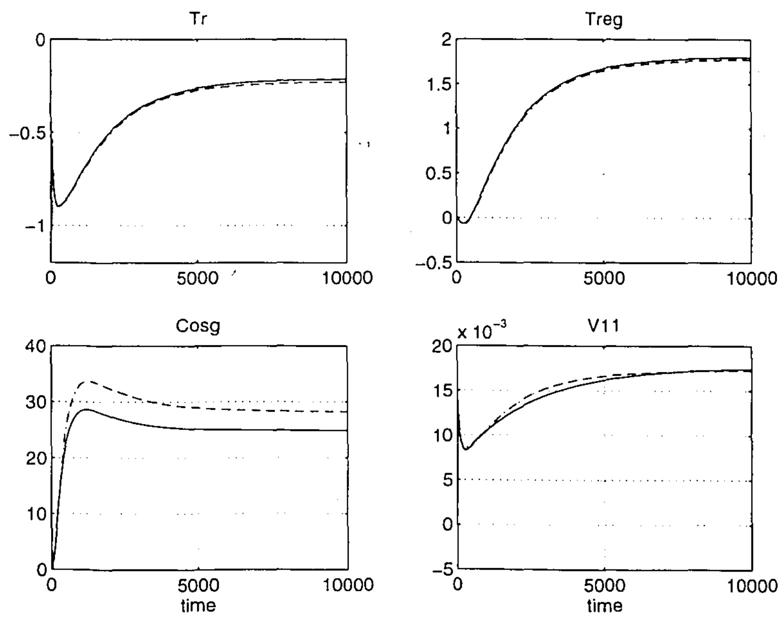


Figure 2.2: Step response in feed for open loop SISO model. Identified model(solid), Actual response(dot-dash)

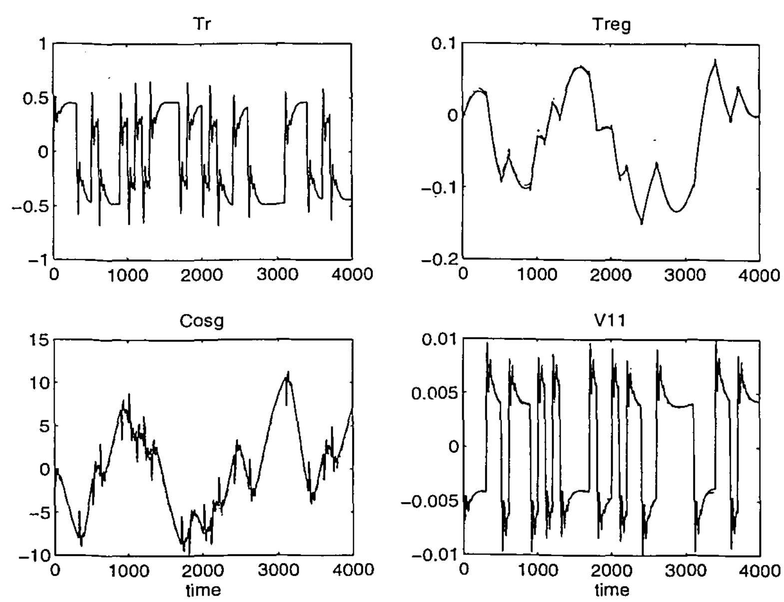


Figure 2.3: Response to PRBS input in feed . SISO open loop model(solid), Actual response(dot-dash)

A straightforward analysis of the frequency domain bias expression for the closed-loop identification criterion given by Eq 2.15 provides some useful insights into this issue. The closed-loop transfer functions $S(z)$ and $S(z)C(z)$ act as weights on the parameter estimation problem just as a prefilter $L(z)$. These are functions of the true process and the specific nature of these transfer functions determines the frequency ranges that are attenuated or amplified. In the case of injecting the signal u_d, the transfer function weighting the effect of $u_d(t)$ is the sensitivity function $S(z)$. If the external signal is introduced at the set point i.e. the signal $r(t)$ is used for the excitation, the signal is weighted by $S(z)C(z)$. These closed-loop transfer functions globally weigh the bias $P - \hat{P}$ in the identification criterion for both the DI and TS methods.

To illustrate the difference between the two locations, the closed-loop transfer functions for a known linear system with a digital implementation of a PI controller was studied.

$$G_p(z) = \frac{8.96z^2 - 11.81z^1 + 3.69}{z^4 - 2.38z^3 + 1.88z^2 - 0.49z} \quad G_c(z) = \frac{k_c}{\alpha}\frac{z - \alpha}{z - 1}$$

$$\text{whereas } \alpha = \frac{\tau_I}{(T_s + \tau_I)}$$

Fig. 2.4 shows the frequency response of the known linear system and the controller for 4 sets of tuning parameters. The sensitivity function $S(z)$ and the transfer function $S(z)C(z)$ for the four different controller tunings are shown in Fig 2.5 and 2.6. As shown in the Fig 2.5, for controllers with $K_c = 0.1, \tau_I = 20$, the sensitivity function tends to 0 as ω approaches zero. The dip in the function $S(z)$ is far more, per decade of frequency, then that of $S(z)C(z)$. This means that the low frequency information of the signal injected at u_d is attenuated due to the control action and the resulting model has better fit in the higher frequency range. Hence the external signal should be injected at setpoint because the weighting function and will result in smaller bias for lower frequency. Also from Fig 2.6, for controllers with $K_c = 0.001, \tau_I = 20$, the change in amplitude plot is almost similar for $S(z)$ and $S(z)C(z)$. But the frequency function $S(z)C(z)$ dips at higher frequency, whereas $S(z)$ is almost flat for $\omega > 0.01 rad/s$. Since we would like to have an accurate estimate of the the ultimate frequency of the system, which is around 0.1 rad/sec, it

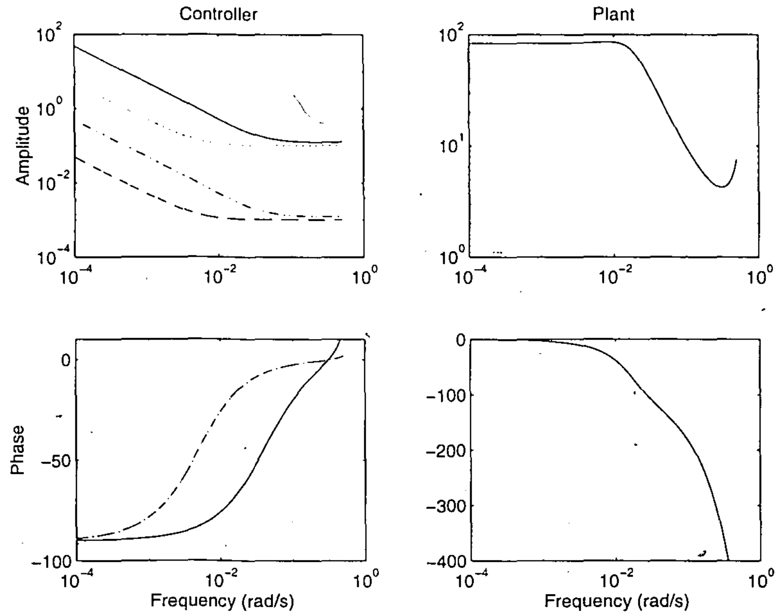


Figure 2.4: Frequency response the known linear system and the controller for 4 controller tuning. $K_c = 0.1, \tau_I = 20$ (solid), $K_c = 0.001, \tau_I = 20$ (dotted) , $K_c = 0.1, \tau_I = 200$(dash-dotted), $K_c = 0.001, \tau_I = 200$ (dash-dotted)

would be advisable to inject the signal at process input because the weighting function in that case,$S(z)$, is almost flat. The Fig. 2.5 and 2.6 thus illustrates that the controller tuning can play an important role in deciding the injection point in the closed loop. Similar trends were observed even when τ_I was increased to 200. and increases with increasing frequency. The results have been summarized in Table. 5.1

Case	Controller Settings		Location of Signal	
	K_c	τ_I	**Signal Injection**	**Weighting Fn.**
Case I	0.1	20	Setpoint	$C(z)S(z)$
Case II	0.1	200	Process Input	$S(z)$
Case III	0.001	20	Setpoint	$C(z)S(z)$
Case IV	0.001	200	Process Input	$S(z)$

Table 2.1: Signal injection in closed loop for four different controller tuning

In practice, however, the nature of these closed-loop transfer functions are not

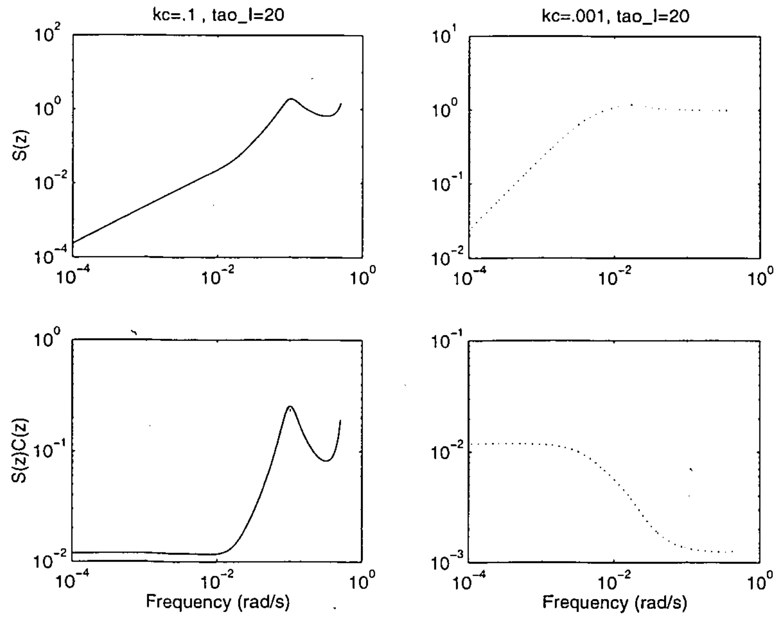


Figure 2.5: Frequency response of closed loop weighting function $S(z)$ and $C(z)S(z)$ for different controller settings. Controller tuning $K_c = 0.1, \tau_I = 20$(solid), $K_c = 0.001, \tau_I = 20$(dotted)

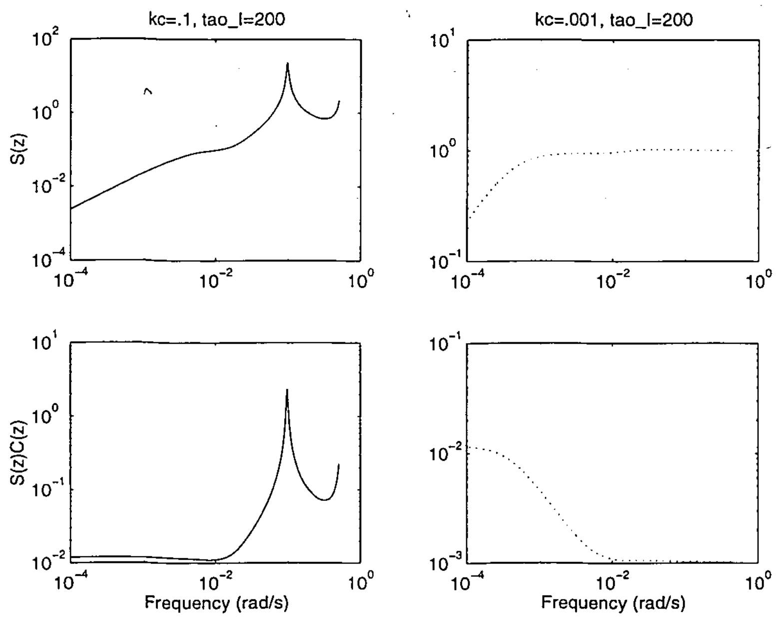


Figure 2.6: Frequency response of closed loop weighting function $S(z)$ and $C(z)S(z)$ for different controller settings. Controller tuning $K_c = 0.1, \tau_I = 200$(solid), $K_c = 0.001, \tau_I = 200$(dotted)

known a priori and the external signal design should be based on some prior knowledge about the process. Just as in the case of open-loop identification, the whole exercise is an iterative procedure. However, some important general conclusions can be made by examining the nature of the amplitude plots of the closed-loop transfer functions in Figure 2.4. Even though, the real nature of these transfer functions is not known, their relative behavior is completely determined by the controller. If the controller parameters are chosen such that the amplitude of the controller transfer function is less than one for all frequencies, then the magnitude of $S(z)C(z)$ is always less than that of S irrespective of the actual nature of S. This implies that a signal injected at the set point, r is attenuated as compared to the signal introduced at the manipulated variable, u_d. On the other hand, if the controller tuning is such that amplitude of C is always greater than one, then the signal at the set point, r is amplified as compared to u_d. This type of behavior may have important implications on the identification of processes which are highly nonlinear with respect to the input magnitude. In this case, depending on the nature of the particular controller employed during the identification experiment, the external signal may result in a very high magnitude of the process input and influence the identification results due to to the presence of process nonlinearities. Similarly, the identification results may be affected in the presence of measurement noise and unmeasured disturbances also, since the signal to noise ratio will be influenced by the magnitude of the input variable.

Closed Loop Identification : FCCU Simulation

The excitation signal was injected at u_d for the identification. Form the preliminary knowledge of the sensitivity function and controller tuning it was found that the signal will be attenuated hence the input amplitude was increased to over come the effect of noise in the input signal. A digital version of the continuous time digital PI was used. Controlled setting was determined from the Z-N rule. The knowledge of ultimate gain K_u and ultimate frequency w_u was obtained from open loop data. Simulations were carried out with random noise in the coking factor.

The effect of noise on the identified model for TS Method and DI is discussed here. First simulation was carried out with low noise to signal ratio whereas the

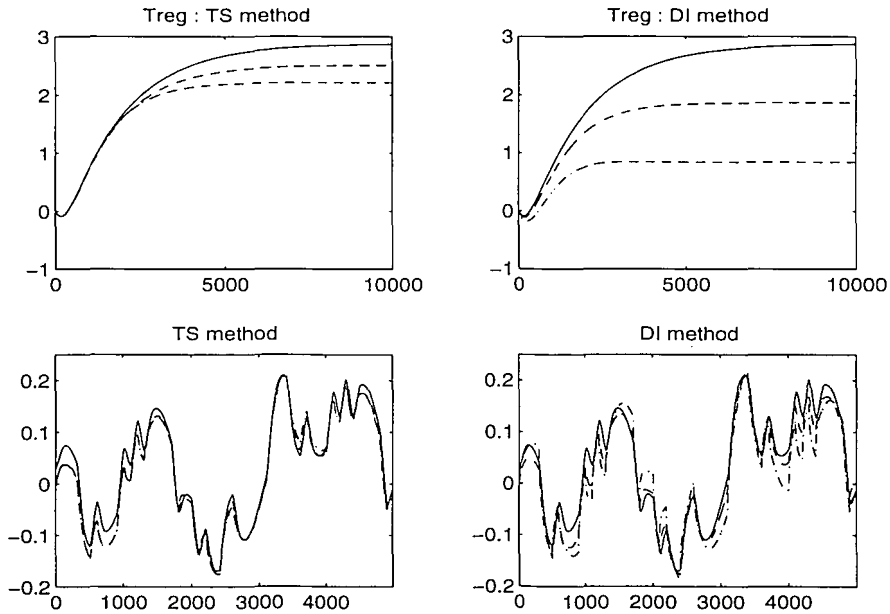


Figure 2.7: Step response and the simulated output for transfer function between Treg and Liftair identified using TS and DI for closed loop operation. Open loop SISO model(solid), High noise to signal ratio (dot-dash), Low noise to signal ratio (dash)

second set of data was generated using higher noise contribution in the signal. From Fig 2.7 it can be observed that step response of the model identified using TS method has almost similar dynamics for both the cases, unlike DI. For direct identification we has similar results as compared to TS method for less noisy signal, which is obvious because the bias introduced in the model will be small as the disturbance spectrum is not a dominating term in the frequency weighting term. Whereas we find that for noisy signal their is a deterioration in the performance of the model identified using DI. So the example illustrates the fact that TS method is able to remove the correlation between the process input and the noise by reconstructing the noise free signal. The TS method will perform better than DI for closed loop data particularly for the case of high noise to signal ratio.

Simulation using TS method with Liftair as a manipulated variable is also discussed in this section. Fig. 2.8 shows the frequency response of the identified model between Liftair and V11. To get some insight about the bias of the identified model frequency response of the closed loop sensitivity function is also given in Fig. Comparing with the actual frequency response of the system it can be seen that identified model does a reasonable job of identifying the high frequency component of the system . But the steady state fit was not very accurate, this is because of the attenuation at lower frequency due to the nature of the sensitivity function (seen in Figure 2.8).

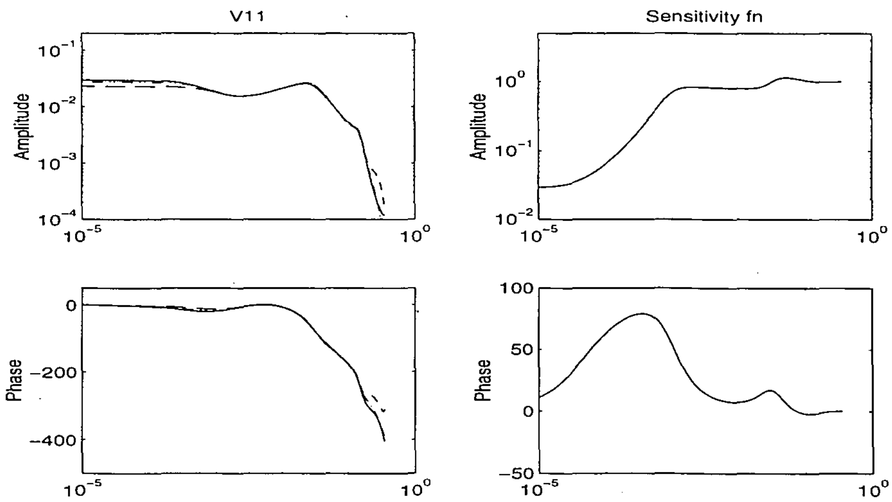


Figure 2.8: ~~Frequency response of the Liftair-~~V_{11} pair and the frequency response of the closed loop sensitivity function. Open loop SISO model(solid), Two step method SISO Closed loop (dot-dash)

2.5 Conclusion

In this chapter we have treated Prediction Error Method for open loop and closed loop identification. Important conclusions regarding the bias in the closed-loop case was obtained form the open-loop analysis. Frequency domain expression was analysed in view of model parameterization to indicate the accuracy of the identified model in various frequency range. It was pointed out that various parameterization like ARX or OE, has different effects on the model fit. Optimization criteria for Two Step method suggests that error in the identified model is due to the modeling error from both the steps. But the consistent estimate of closed-loop sensitivity function *is not mandatory for good model fit in the second step. This indicates that we could afford higher bias in the estimation of sensitivity function for the frequency range where it is low in magnitude, and still estimate a good plant model for that frequency. Example for SISO cases were presented and it has been shown the PEM methods work well for these cases. A known linear system was studies to develop some guidelines about the injection of signal in closed loop. It was indicated that signal should be injected at the location which results in weighting function which is flat in the desired frequency range. It was pointed out that the controller tuning plays an important role in deciding the signal injection in closed loop. Simulation results on FCCU indicates that the TS method results in better models as compared to DI, particularly for high noise to signal ratio. Simulation studies indicates that the bias in the identified model was high for the frequency range for which the closed loop sensitivity function was low in magnitude.*

Results for multivariable system are not discussed here. We feel that comparative study between various algorithm will be more beneficial in understanding the advantages and disadvantage between them. The theory for one of the multivariable algorithm is discussed in next few chapters, hence we have treated the multivariable problem somewhere latter in the thesis

Chapter 3

Open Loop Subspace Identification Algorithm

This chapter provides the mathematical foundation required to build up the subspace identification algorithm. In Sec 3.1 some preliminary definition regarding vector and vector space is discussed. Subspace identification algorithm are often based on geometric concepts. As it is shown in this chapter, the system characteristics can be revealed form the geometric manipulation of certain matrices. Sec. 3.2 uses the matrices to define a vector space and subsequently the matrix projection namely Orthogonal and Oblique projection. Instead of presenting them in system identification framework, they are described from linear algebra point of view and hence they can be applied directly in any other development. Efforts are made in Sec 3.3 to analyze the matrix projection in terms of basis for vector space. In Sec. 3.4 we discuss the classical and recent development in subspace identification. The Sec 3.4.3 describe the deterministic algorithm, whereas Sec. 3.4.4 details the deterministic-stochastic identification algorithm of Vanoverschee (1995)

3.1 Vectors and Vector Space

A vector is an element of a linear vector space $\mathcal{S}$ that satisfies some requirements. A vector space $\mathcal{S}$ is the set $\mathcal{S}$ whose members satisfy the following criteria:

- An operation denoted by + , called addition, is defined in such a way that for all x and y that are element of $\mathcal{S}$, the operation $x+y$ results in a another element which is also a part of $\mathcal{S}$. Furthermore addition should be commutative and associative.

$$\begin{aligned} \forall \quad x, y \in \mathcal{S} \quad &\Rightarrow x+y \ = y+x \in \mathcal{S} \\ \forall \quad x, y, z \in \mathcal{S} \quad &\Rightarrow (x+y)+z \ = x+(y+z) \in \mathcal{S} \end{aligned} \tag{3.1}$$

- $\mathcal{S}$ contains a null vector, $\emptyset$ such that for all x that are element of $\mathcal{S}$, addition of the element to null vector will result in the original element itself.

$$\exists \text{ a zero vector}, \emptyset \in \mathcal{S} \quad \ni x+\emptyset = x \quad \forall x \in \mathcal{S} \tag{3.2}$$

- For each x in $\mathcal{S}$ there is a vector "$-x$" such that addition operation between the two will result in a null vector $\emptyset$.

$$\forall \quad x \in \mathcal{S} \quad \Rightarrow \quad \exists \text{ a vector } -x \quad \ni x+(-x) = \emptyset \tag{3.3}$$

- An operation called scalar multiplication is defined in such a way that for all the x that are element of $\mathcal{S}$ and the scalar α the operation αx results in a vector which is also a part of $\mathcal{S}$

$$\forall\, x \in \mathcal{S} \; and \, \forall\, \alpha, \beta \in \Re \quad \Rightarrow \quad (\alpha+\beta)x = \alpha x + \beta x \in \mathcal{S} \tag{3.4}$$

The set $M = \Re^{n \times m}$ containing all $n \times m$ matrices, satisfies all the properties of vector space, hence M can be thought as an element of this vector space. We are familiar with the projection of one vector on to another. Since matrices can be treated as members of a vector space we can likewise define matrix projection, which refers to the fact that a matrix can be decomposed along two different matrices. This fact is discussed in detail in the next few sections.

For the following few section we assume that Matrix $A \in \Re^{p \times j}, B \in \Re^{q \times j}, C \in \Re^{r \times j}$. The element of a row of one of the given matrices can be considered as the coordinates of a vector in the j-dimensional vector space. The matrix projection can be interpreted in terms of the column space [1] or the row space [2] of a matrix. The projection of row space of A on to the row space of B can be interpreted as the decomposition of rows of A as the linear combination of the rows of B. Similarly projection of column space of one matrix on the another is nothing but decomposing the columns of a matrix in terms of linear combination of columns of another matrix. In the following few sections we define two types of matrix projection, Orthogonal and Oblique , which form the basis for the subspace identification technique.

3.2 Geometric Tools : Matrix Projection

3.2.1 Orthogonal Projection

The projection of row space of matrix A on to row space of B represents a matrix which has the rows that are the least square solution of rows of A, where the rows of B and its orthogonal complement defines the vector space. If the row space of A and B are the same, then it is implied that the rows of A can be expressed exactly as the linear combination of rows of B. So for the case when the row space of both the matrices are the same, the projection of A on B will result in the matrix A. Whereas if the row space of both the matrices are orthogonal to each other the matrix projection will result in a null matrix.

[1] If a vector space S consists of all the linear combinations of the particular vectors $e_1, e_2, \cdots, e_n$, then these vectors **span** the space. In other words, every vector v in S can be expressed as some combination of e_j *Column space:* Is the space spanned by the columns of the given matrix i.e it is the vector space formed by the linear combination of all the columns of a matrix.

[2] *Row space:*. Is is the vector space generated by the linear combination of all the rows of a given matrix. For a matrix the number of linearly independent rows is equal to the number of linearly independent columns, hence row space and the column space are of same dimension and is equal to the rank of the matrix.

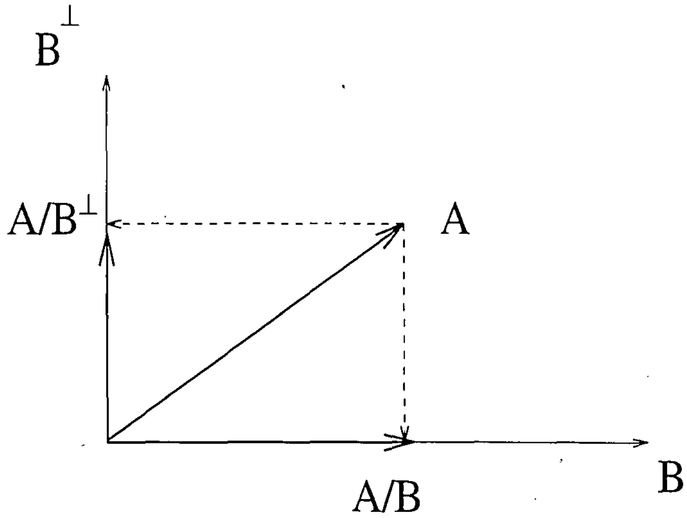


Figure 3.1: Orthogonal Projection of a matrix A on to the row space of B and its orthogonal complement $B^{\perp}$

If Π_B denotes the operator that projects the row space of a matrix onto the row space of the matrix $B \in \Re^{q \times j}$ then the operator can be interpreted as [3]

$$\Pi_B = B^T \left(BB^T\right)^{\dagger} B \tag{3.5}$$

For the definition of the projection operator given by 3.5 we can define the projection of $A \in \Re^{p \times j}$ on $B \in \Re^{q \times j}$ as

$$A/B \ = \ A\Pi_B \ = \ AB^T \left(BB^T\right)^{\dagger} B \tag{3.6}$$

If $\Pi_{B^{\perp}}$ denotes the operator that projects the row space of a matrix onto the orthogonal component of row space of the matrix $B \in \Re^{q \times j}$ then the operator can be interpreted as 3.7 and the projection of $A \in \Re^{p \times j}$ on the orthogonal component of $B \in \Re^{q \times j}$ can be given as 3.8

$$\Pi_{B^{\perp}} = I_j - \Pi_B \tag{3.7}$$

$$A/B^{\perp} = A\Pi_{B^{\perp}} \tag{3.8}$$

[3]$\dagger$ represents the pseudo-inverse of a matrix

Moreover the combination of the projection Π_B and $\Pi_{B^\perp}$ can be used to decompose the matrix A into two matrices of which the row spaces are orthogonal.

$$A = A\Pi_B + A\Pi_{B^\perp} \tag{3.9}$$

In other words the projection decomposes the row space of A as the linear combination of the rows of B and those of the orthogonal component of B.

$$A = L_B B + L_{B^\perp} B^\perp \tag{3.10}$$

where $B^\perp$ represents the orthogonal component of the row space of B.

$$L_B B = A\Pi_B \tag{3.11}$$

$$L_{B^\perp} B^\perp = A\Pi_{B^\perp} \tag{3.12}$$

3.2.2 Oblique Projection

Generalizing the concept of orthogonal projection we can define another matrix projection namely *Oblique Projection*, which is used extensively in the subspace identification algorithm. Instead of projecting the row space of A on to B and then decomposing it as a linear combination of B and $B^\perp$, we can project it onto the combined row space of B and C and then represent it in terms of B, C and $B^\perp, C^\perp$. Moreover the projection on to the combined row space of B and C can be decomposed along B and C separately, which we refer as an oblique projection.

The matrix A can be written in terms B, C and orthogonal component of $B^\perp, C^\perp$ as

$$A = L_{B,C} \begin{pmatrix} B \\ C \end{pmatrix} + L_{B^\perp,C^\perp} \begin{pmatrix} B \\ C \end{pmatrix}^\perp \tag{3.13}$$

where the component along B, C (compound projection) can be expressed as 3.14. The oblique projection,$L_C C$, is the component of the compound projection

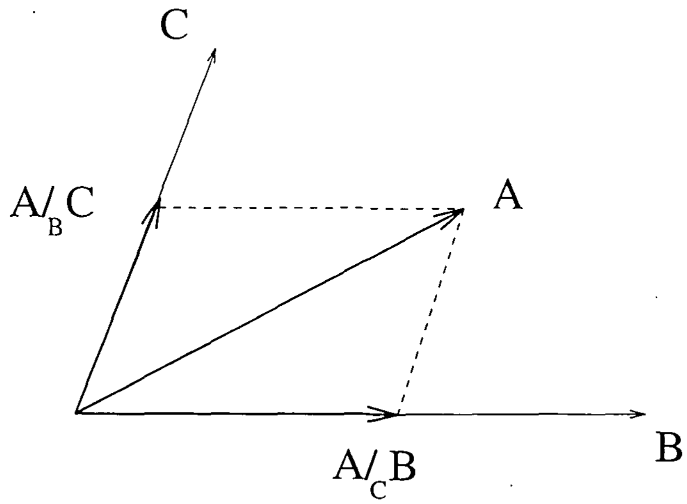


Figure 3.2: Oblique Projection of matrix A on to the combined row space of B and C

along the matrix C. In other words it is the projection of row space of A along the rowspace of B on the row space of C. Similarly the term $L_B B$ represent the projection of row space of A along the rowspace of C on the row space of B

$$L_{B,C}\begin{pmatrix} B \\ C \end{pmatrix} = L_B B + L_C C \tag{3.14}$$

$$A/_B C = L_C C \tag{3.15}$$

$$A/_C B = L_B B \tag{3.16}$$

Using Eq. 3.6 we can write the projection on the compound row space of B, C as

$$A/\begin{pmatrix} C \\ B \end{pmatrix} = A(C^T\ B^T)\begin{pmatrix} CC^T & CB^T \\ BC^T & BB^T \end{pmatrix}^{\dagger}\begin{pmatrix} C \\ B \end{pmatrix} \tag{3.17}$$

According to (Van Overschee 95) the oblique projection can be calculated as

$$A/_B C = A(C^T\ B^T)\left[\begin{pmatrix} CC^T & CB^T \\ BC^T & BB^T \end{pmatrix}^{\dagger}\right]_{\text{first r columns}} C \tag{3.18}$$

It can be also be represented terms of Orthogonal Projection as

$$A/_B C = \left[A/B^{\perp}\right]\left[C/B^{\perp}\right]^{\dagger} C \tag{3.19}$$

$$A/_C B = \left[A/C^{\perp}\right]\left[B/C^{\perp}\right]^{\dagger} B \tag{3.20}$$

Here are some of the properties of Oblique Projection which gives the more insight regarding the connecting between Oblique and Orthogonal Projection.

1. If $B = 0$ or $BC^T = 0$ (B, C are orthogonal to each other) the oblique projection reduces to

$$\begin{aligned} A/_B C &= A/C \\ A/_C B &= A/B \end{aligned} \tag{3.21}$$

2. Projection of B along B on C results in null matrix

$$\begin{aligned} B/_B C &= 0 \\ C/_C B &= 0 \end{aligned} \tag{3.22}$$

3. Projection of C along B on C results in C matrix itself

$$\begin{aligned} C/_B C &= C \\ B/_C B &= B \end{aligned} \tag{3.23}$$

3.3 Norm and Inner Product

To make the vector space complete one has to be provide it with the sense of length and direction. The full significance of addition and multiplication operation, defined earlier in this chapter, can be realized only under this complete definition. For a vector we define an operator, *norm*, which gives us the length of a vector. We introduce an operator, *inner product*, such that it gives some sense of direction. The norm and inner product of a vector space has to satisfy some requirements. Moreover one can ask, for the matrix projection defined in the previous section, what is the definition of norm and inner product.

3.3.1 Definition for Norm and Inner Product

The norm, $\|.\|$, should have following properties for all $x \in S$

1.

$$\|\alpha x\| = |\alpha| \|x\| \tag{3.24}$$

2. Positiveness :

$$\begin{aligned} \|x\| &> 0 \quad \forall\, x \neq \emptyset \\ \|x\| &= 0 \quad \forall\, x = \emptyset \end{aligned} \tag{3.25}$$

3. Triangle inequality

$$\|x + y\| \leq \|x\| + \|y\| \tag{3.26}$$

The inner product between two vectors, (x, y), should satisfy the conditions given below.

1. Conjugate Symmetry:

$$(x, y) = \overline{(y, x)} \tag{3.27}$$

2. Linearity :

$$(\alpha x + \beta y, z) = \alpha(x, z) + \beta(y, z) \tag{3.28}$$

3. Positiveness

$$\begin{aligned} (x, x) &> 0 \quad \forall\, x \neq \emptyset \\ (x, x) &= 0 \quad \forall\, x = \emptyset \end{aligned} \tag{3.29}$$

3.3.2 Basis for Vector Space

Furthermore a vector set S is *n dimensional* if it contains a set of n linearly independent vectors. If $e_1, e_2, \cdots, e_n$ form the basis for the given vector space then any n-dimensional vector can be represent in terms of linear combination of them.

$$x = \alpha_1 e_1 + \alpha_2 e_2 + \cdots + \alpha_n e_n$$
$$\text{whereas} \qquad \alpha_j = \frac{(x, e_j)}{(e_j, e_j)} \tag{3.30}$$

(x, e_j) and (e_j, e_j) are defined as the corresponding inner product for the vector space.

For the case of Matrix projection we have shown before that $A \in \Re^{p \times j}$ can be decomposed as the linear combination of rows of $B \in \Re^{q \times j}$ and its orthogonal component.

$$A = \alpha_1 B + \alpha_2 B^{\perp}$$
$$\text{where} \qquad \alpha_1 = L_B, \qquad \alpha_2 = L_{B^{\perp}} \tag{3.31}$$

The row of matrix A can be considered as the coordinate of a vector in j - dimensional vector space. If A is of full rank, we require p (here we assume the number of columns are greater that number of rows, hence the rank of the matrix is equivalent to maximum row rank) independent rows (in a j-dimensional space) to decompose the rows of A in terms of other matrix. We find that rows of B and $B^{\perp}$ form a j dimensional space and hence are sufficient to expand any matrix of rank j in the j dimensional subspace. Instead of expanding the rows of matrix in terms of the j independent rows we can use more compact notation and can expand in term of B and $B^{\perp}$. Unlike 3.30, we will have α_j as matrices for this expansion. The coefficient, α_j, can be easily calculated using the definition of inner product and norm as.

$$\alpha_1 = \frac{(A, B)}{(B, B)} \qquad \alpha_2 = \frac{(A, B^{\perp})}{(B^{\perp}, B^{\perp})} \tag{3.32}$$

But for the Eq. 3.31 we know

$$\begin{aligned} L_B &= AB^T\left(BB^T\right)^\dagger \\ L_{B^\perp} &= AB^{\perp T}\left(B^\perp B^{\perp T}\right)^\dagger \end{aligned} \tag{3.33}$$

3.3.3 Example for Orthogonal Projection

Considering the orthogonal projection of matrix A on B, where A, B are defined as

$$\begin{aligned} A &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 3 & 1 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} | & | \\ A_1 & A_2 \\ | & | \end{bmatrix} \\ B &= \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} | & | \\ B_1 & B_2 \\ | & | \end{bmatrix} \end{aligned} \tag{3.34}$$

We can interpret the projection in terms of the column space of B. All the possible linear combination of B_1 and B_2 defines the column space of B. If the column of A are in the column space of B the projection of A on B will result in A itself. But for this example neither of the column of A is in the column space of B, hence the projection will result in the least square solution. The projection on B and the projection on orthogonal component of B is shown in Fig. 3.3

3.4 State Space Identification Problem

Considering a linear time invariant (LTI) system with the state space realization

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned} \quad \text{with}$$

$$E\left[\begin{pmatrix} w_k \\ v_k \end{pmatrix}\begin{pmatrix} w_l^T & v_l^T \end{pmatrix}\right] = \begin{pmatrix} Q_s & S_s \\ (S_s)^T & R_s \end{pmatrix}\delta_{kl} \geq 0 \tag{3.35}$$

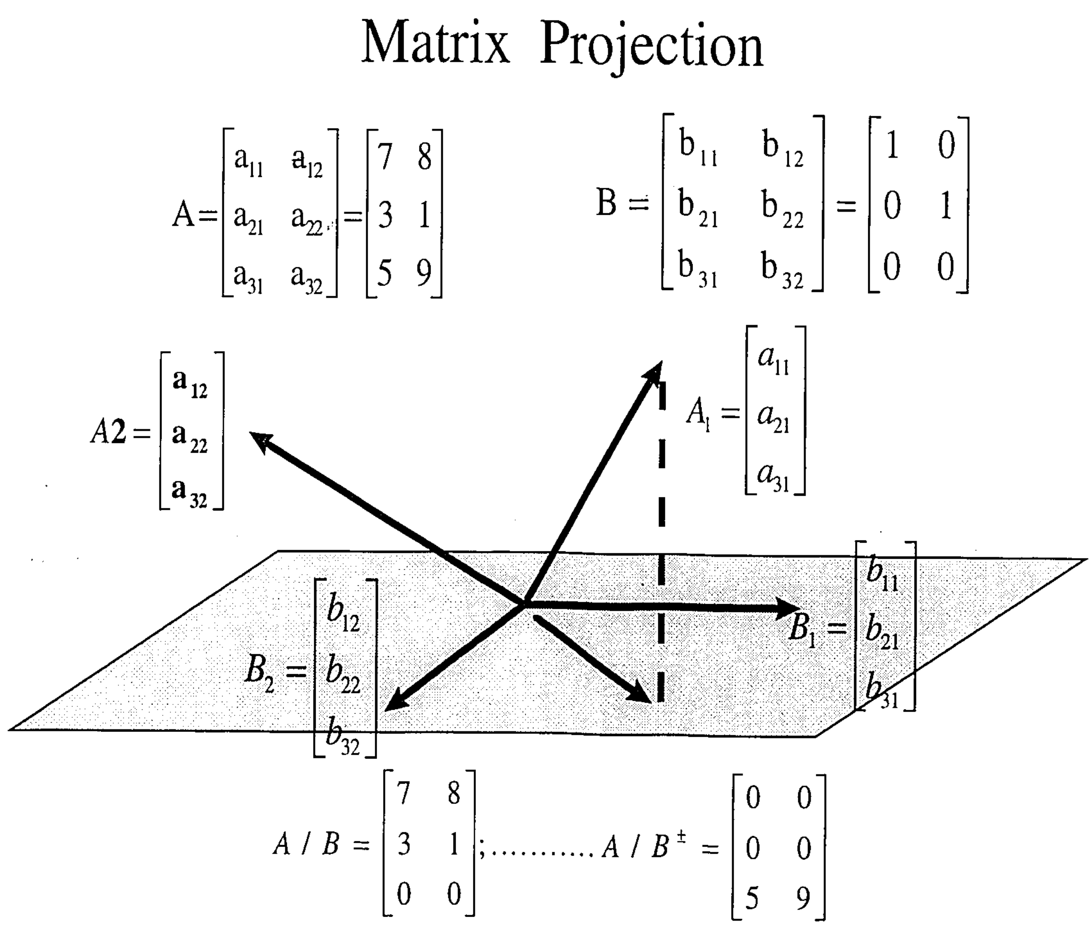


Figure 3.3: Orthogonal Projection of matrix A on to row space of B and its orthogonal complement $B^{\perp}$

Here x_k, y_k, u_k are vectors with dimensionality n, m and l respectively. Also the dimensionality of the matrices involved are $A \in \Re^{n \times n}, B \in \Re^{n \times m}, C \in \Re^{l \times n}$ and$D \in \Re^{l \times m}$. We assume that we have N measurements of inputs u_k and outputs y_k. Furthermore w_k and v_k are assumed to be n and l dimensional zero mean white noise vectors and represents the process and measurement noise respectively.

3.4.1 Classical Realization Theory

In the absence of any noise the input-output relation can be described as

$$y_k = \sum_{k=0}^{\infty} h_k u_{t-k} \text{ where } h_k = \begin{cases} 0 & , \ k < 0 \\ D & , \ k = 0 \\ CA^{k-1}B & , \ k > 0 \end{cases} \tag{3.36}$$

Here $h_k \in \Re^{l \times m}$ is the impulse response matrix of the system. Classical realization theory deals with the problem of finding a minimal state-space model given the impulse response h_k. Constructing the Hankel Matrix of order $(n \times 1) \times (n \times 1)$ as follows

$$H = \begin{bmatrix} h_1 & h_2 & \cdots & h_{n+1} \\ h_2 & h_3 & \cdots & h_{n+2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n+1} & h_{n+2} & \cdots & h_{2n+1} \end{bmatrix} \tag{3.37}$$

it can be easily shown that it can be factorized as

$$H = \Gamma_{n+1}\Omega_{n+1} \quad \text{where } \Omega_n = \left[B, AB, \cdots, A^{n-1}B\right] \ \Gamma_n = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{3.38}$$

Γ_n, Ω_n are the observability and the controllability matrix of the system. The system matrices B,C can be readily calculated from the first column block (m columns) of Ω_n and first row block (l rows) of Γ_n respectively. The A matrix can be calculated from the shift invariant structure of Γ_n

$$A = \Gamma_{2:n+1}^{-1}\Gamma_{1:n} \tag{3.39}$$

3.4.2 Subspace Identification for Open Loop case

For purely deterministic system we have no measurement or process noise $v_k = w_k = 0$. So the state space representation can be written as

$$\begin{aligned} x_{k+1}^d &= Ax_k^d + Bu_k \\ y_k &= Cx_k^d + Du_k \end{aligned} \tag{3.40}$$

where $()^d$ stands for the deterministic part. It is assumed that $\{A, C\}$ is observable while $\{A, B\}$ is controllable.

Preliminaries and Notation

Input output Hankel matrices are defined (Van Overschee, 1995) in terms of process input and output as [4]

$$U_{0|2i-1} \equiv \overbrace{\left[\begin{array}{ccccc} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ u_{i-1} & u_i & u_{i+1} & \cdots & u_{i+j-2} \\ \hline u_i & u_{i+1} & u_{i+2} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & u_{i+3} & \cdots & u_{i+j} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ u_{2i-1} & u_{2i} & u_{2i+1} & \cdots & u_{2i+j-2} \end{array}\right]}^{\text{j columns}}$$

$$\equiv \left(\frac{U_{0|i-1}}{U_{i|2i-1}}\right) \equiv \left(\frac{U_p}{U_f}\right) \quad \text{where} \quad (U_p, U_f \in \Re^{mi \times j}) \tag{3.41}$$

[4] where j is typically equal to $N - 2i + 1$

$$Y_{0|2i-1} \equiv \left[\begin{array}{ccccc} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{i-1} & y_i & y_{i+1} & \cdots & y_{i+j-2} \\ \hline y_i & y_{i+1} & y_{i+2} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \cdots & y_{i+j} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{2i-1} & y_{2i} & y_{2i+1} & \cdots & y_{2i+j-2} \end{array}\right]$$

$$\equiv \left(\begin{array}{c} Y_{0|i-1} \\ \hline Y_{i|2i-1} \end{array}\right) \equiv \left(\begin{array}{c} Y_p \\ \hline Y_f \end{array}\right) \quad \text{where} \quad (Y_p, Y_f \in \Re^{mi \times j}) \tag{3.42}$$

where as $()_p, ()_f$ stands for past and future respectively. Also $()^+, ()^-$ are defined by shifting the border between the past and future data by one block row as

$$\left(\begin{array}{c} U_{0|i} \\ \hline U_{i+1|2i-1} \end{array}\right) \equiv \left(\begin{array}{c} U_p^+ \\ \hline U_f^- \end{array}\right) \quad \text{and} \quad \left(\begin{array}{c} Y_{0|i} \\ \hline Y_{i+1|2i-1} \end{array}\right) \equiv \left(\begin{array}{c} Y_p^+ \\ \hline Y_f^- \end{array}\right) \tag{3.43}$$

Defining the block Hankel matrices consisting of inputs and output as

$$W_{0|i-1} \equiv \begin{pmatrix} U_{0|i-1} \\ Y_{0|i-1} \end{pmatrix} \equiv \begin{pmatrix} U_p \\ Y_p \end{pmatrix} \equiv W_p \quad \text{where} \quad \left(W_p \in \Re^{(m+l)i \times j}\right) \tag{3.44}$$

$$W_{i|2i-1} \equiv \begin{pmatrix} U_{i|2i-1} \\ Y_{i|2i-1} \end{pmatrix} \equiv \begin{pmatrix} U_f \\ Y_f \end{pmatrix} \equiv W_f \quad \text{where} \quad \left(W_f \in \Re^{(m+l)i \times j}\right) \tag{3.45}$$

$$W_p^+ \equiv \begin{pmatrix} U_p^+ \\ Y_p^+ \end{pmatrix} \quad W_f^- \equiv \begin{pmatrix} U_f^- \\ Y_f^- \end{pmatrix} \tag{3.46}$$

The deterministic state sequence X_i^d is defined as

$$X_i^d = \left(x_i^d, x_{i+1}^d, \cdots, x_{i+j-2}^d, x_{i+j-1}^d\right) \quad \left(x_i^d \in \Re^{n \times 1}, X_i^d \in \Re^{n \times j}\right) \tag{3.47}$$

Here i denotes the subscript of the first element of state sequence. Similar to the definition of past inputs and outputs we denote past and future states as

$$X_p^d = X_0^d \ , X_f^d = X_i^d \tag{3.48}$$

We define the extended observability matrix Γ_i and reversed extended controllability matrix Δ_i as

$$\Delta_i = \left[A^{i-1}B, A^{i-2}B, \cdots, AB, B\right] \ \Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix}$$
$$\text{where } \left(\Delta_i \in \Re^{nxmi}, \Gamma_i \in \Re^{lixn}\right) \tag{3.49}$$

and the lower block triangular Teoplitz matrix H_i^d as

$$H_i^d = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \cdots & D \end{bmatrix} \quad \left(H_i^d \in \Re^{lixmi}\right) \tag{3.50}$$

It can be shown that under these definition (De Moor, 1988) the state space equation can be written in matrix input-output equation as

$$\begin{aligned} Y_p &= \Gamma_i X_p^d + H_i^d U_p \\ Y_f &= \Gamma_i X_f^d + H_i^d U_f \\ X_f^d &= A^i X_p^d + \Delta_i U_p \end{aligned} \tag{3.51}$$

3.4.3 Subspace algorithm for Deterministic case

We define Θ_i as the oblique projection of Y_f along U_f on W_p as

$$\Theta_i = Y_f /_{U_f} W_p \quad \text{also} \quad \Theta_{i-1} = Y_f /_{U_f^-} W_p^+ \tag{3.52}$$

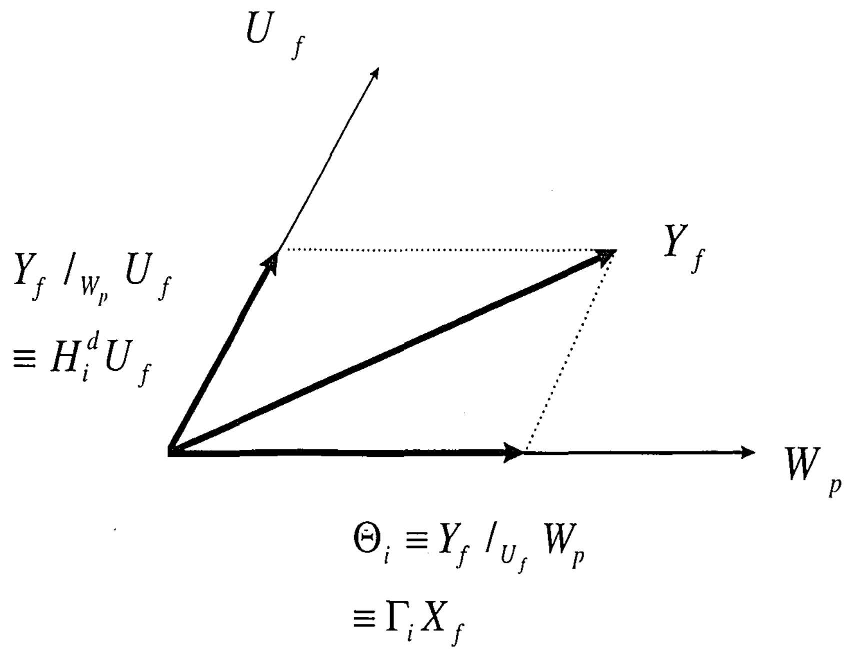


Figure 3.4: Oblique Projection for open loop deterministic subspace identification algorithm. The oblique projection decomposed the future output Y_f along the future input U_f and the past input-output data W_p

Singular value decomposition of Θ_i results in

$$\begin{aligned}\Theta_i &= (U_1 \;\; U_2)\begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \\ &= U_1 S_1 V_1^T \end{aligned} \tag{3.53}$$

Moreover it can be proved that Θ_i can be represented in terms of Γ_i and X_f^d (Van Overschee, 1995) as

$$\Theta_i = \Gamma_i X_f^d \tag{3.54}$$

States of the system X_i^d and the system matrices A, B, C, D can be calculated as by the knowledge of Γ_i, whereas $\underline{\Gamma_i}$ represents Γ_i without the last l rows.

$$\begin{aligned} &\Gamma_i = U_1 S_1^{1/2} \;\; , \;\; \Gamma_{i-1} = \underline{\Gamma_i} \\ &X_i^d = \Gamma_i{}^{\dagger}\Theta_i \;\; , \;\; X_{i+1}^d = \Gamma_{i-1}{}^{\dagger}\Theta_{i-1} \end{aligned} \tag{3.55}$$

Once the state of the system is calculated the solution of system matrices is obtained by solving the least square problem.

$$\begin{pmatrix} X_{i+1}^d \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} X_i^d \\ U_{i|i} \end{pmatrix} \tag{3.56}$$

3.4.4 Subspace Algorithm for Deterministic-Stochastic Case

Most of the real life measurement have some measurement or process noise. Hence the question of asymptotic unbiasedness , i.e given an infinite amount of noisy data , is critical for the success of any identification algorithm. The algorithm present in the Sec can not be applied to this case because the identified state sequence is biased for finite data length and hence the identified transfer function will have some bias. So the deterministic algorithm has to be modified a bit to get the estimate of the actual state sequence from the input-output data. This section present the algorithm for the deterministic-stochastic case, but it does not discusses it in details. Reader should refer to (Vanoverschee, 95) for the detailed analysis of the algorithm. Step by step procedure for the algorithm for combined case is:.

- Calculate the oblique and orthogonal projection

$$\Theta_i = Y_f/_{U_f}W_p, \quad Z_i = Y_f/\begin{pmatrix} W_p \\ U_f \end{pmatrix}$$

$$Z_{i+1} = Y_f^-/\begin{pmatrix} W_p^+ \\ U_f^- \end{pmatrix} \tag{3.57}$$

- Calculate the SVD of the oblique projection

$$\Theta_i = (U_1 \ \ U_2)\begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}$$

$$= U_1 S_1 V_1^T \tag{3.58}$$

- Determine Γ_i and Γ_{i-1} as

$$\Gamma_i = U_1 S_1^{1/2} \ , \ \ \Gamma_{i-1} = \underline{\Gamma_i} \tag{3.59}$$

- Solve the linear set of equations

$$\begin{pmatrix} \Gamma_{i-1}^{\dagger} Z_{i+1} \\ \mathbf{Y}_{i|i} \end{pmatrix} = \begin{pmatrix} A & \mathcal{K}_{11} \\ C & \mathcal{K}_{12} \end{pmatrix}\begin{pmatrix} \Gamma_i^{\dagger} Z_i \\ \mathbf{U_f} \end{pmatrix} + \begin{pmatrix} \sigma_w \\ \sigma_v \end{pmatrix} \tag{3.60}$$

- Solve for B and D from the knowledge of $A, C, \mathcal{K}_{11}, \mathcal{K}_{12}$ by minimizing the optimization criterion. (Van Overschee and De Moor, 1994c)

- Determine as Q, S, R from the residual as

$$\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} = E_j\left[\begin{pmatrix} \sigma_w \\ \sigma_v \end{pmatrix}\begin{pmatrix} \sigma_w^T \ \sigma_v^T \end{pmatrix}\right] \tag{3.61}$$

3.5 Conclusion

In this chapter we have introduced the concept that matrices satisfies all the property of a vector space and hence can be used as a vector. The geometric tools like *matrix projection have been presented and it has been shown that certain projection leads to algorithm which helps in revealing the system properties in state space framework. Finally the algorithms for open loop identification for deterministic and deterministic-stochastic case was presented in the framework of matrix projection.*

There are many more interesting subspace identification theory. But we have described only one of them here because it forms the basis for the algorithm which has been developed for closed loop case. The closed loop identification algorithms are detailed in the next chapter. Most of subspace identification algorithm vary in there choice of weighting matrices. An excellent overview of these algorithm has been presented in Van Overschee(95)

Chapter 4

Closed Loop Subspace Identification Algorithm

In previous chapter we presented the subspace algorithm for open loop data. The main focus of this chapter is to develop subspace based state space identification algorithm for closed loop data. Section 4.1 introduces the basic framework of the problem to be solved in this chapter. The section presents the closed loop system dynamics in the state space framework. The closed loop system dynamics consists of plant and controller states. Section 4.1.1 and 4.1.2 details the procedure for extracting plant and controller states respectively from the overall closed loop system. This forms the basis for the closed loop identification algorithm in practice. Section 4.2 describes the step by step procedure to apply the developed algorithm. A technique to guarantee the stability of the identification algorithm has also been proposed. Section 4.2.3 describes the derivation which was done for this purpose. Section clsim1 describes the simulation studies on a fourth order SISO system to prove the efficiency of the developed algorithm. Section 4.3.2 summarizes the results which were obtained by performing the identification with proposed stability modification.

4.1 Introduction: Problem Description

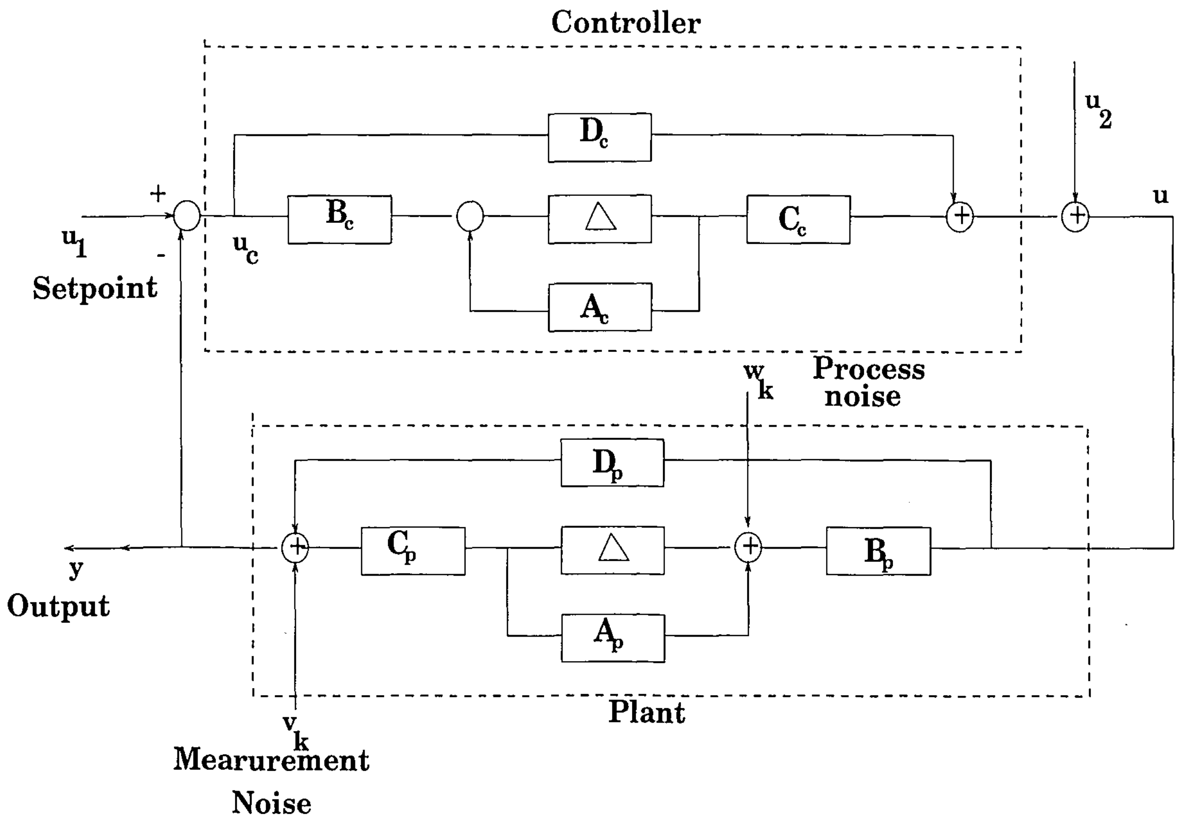


Figure 4.1: Closed-loop system configuration for identification

In this section we show that the plant transfer function can be obtained by posing the closed loop problem as a joint input-output one. The output for the identification algorithm is an augmented vector consisting of plant inputs and outputs. Intuitively it can be said that controller transfer function can also the identified separately because of the symmetry of closed loop configuration. It has been proved in this section that joint input-output identification (with the output defined as the vector consisting of controller input and output) results in the identification of controller transfer function too. Since u_c, y_c can be obtained by algebraic manipulation of u_p, y_p, we expect that both the joint input-output identifications are similar in their underlying principle. This leads us to the result that one identification is sufficient to calculated both the transfer function separately. We are interested in evaluating the plant and controller transfer function from the input-output data. For the assumed closed loop configuration (Fig 4.1) we can have the following input-output measurements

1. u_p, y_p plant input and output respectively
2. u_c, y_c controller input and output respectively
3. u_1, u_2 external signal : setpoint and process input.

For the given closed loop configuration the state space model for plant and controller can be written as

$$\text{Plant :} \quad \begin{aligned} x_p(k+1) &= A_p x_p(k) + B_p u(k) + w_k \\ y(k) &= C_p x_p(k) + D_p u(k) + v_k \end{aligned} \tag{4.1}$$

$$\text{Controller :} \quad \begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c u_c(k) \\ y_c(k) &= C_c x_c(k) + D_c u_c(k) \end{aligned} \tag{4.2}$$

It has been proved in appendix A that plant and controller states can be written in terms of external signal (process input and setpoint). Also it has been proved that the plant input and output can be expressed in terms of overall states and the external signal as

$$\begin{aligned} \begin{bmatrix} x_p^{k+1} \\ x_c^{k+1} \end{bmatrix} &= \begin{bmatrix} A_p - B_p D_c K C_p & B_p(C_c - D_c K D_p C_c) \\ -B_c K C_p & A_c K D_p C_c \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} \\ &+ \begin{bmatrix} B_p D_c K & B_p - B_p K D_p \\ B_c K & -B_c K D_p \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} I & -B_p D_c K \\ 0 & -B_c K \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \end{aligned} \tag{4.3}$$

$$\begin{aligned} \begin{bmatrix} u \\ y \end{bmatrix} &= \begin{bmatrix} -D_c K C_p & C_c - D_c K D_p C_c \\ C_p - D_p D_c K C_c & D_p(C_c - D_c K D_p C_c) \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} \\ &+ \begin{bmatrix} D_c K & I - K D_p \\ D_p D_c K & D_p(I - K D_p) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & -D_p D_c K + I \\ 0 & -D_c K \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \end{aligned} \tag{4.4}$$

For most of the cases of interest we expect the $D_p = 0$. Hence $K = (I + D_p D_c)^{-1} = I$. So the equation simplifies to the following form.

$$\begin{bmatrix} x_p^{k+1} \\ x_c^{k+1} \end{bmatrix} = \begin{bmatrix} A_p - B_p D_c C_p & B_p C_c \\ -B_c C_p & A_c \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} B_p D_c & B_p \\ B_c & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} I & -B_p D_c \\ 0 & -B_c \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{4.5}$$

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} -D_c C_p & C_c \\ C_p & 0 \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} D_c & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & I \\ 0 & -D_c \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{4.6}$$

Moreover we can write a overall state space model between the external signal and the augmented output, consisting of controller ,u, and plant y, outputs as

$$x^{k+1} = A' x^k + [B_1' \;\; B_2'] \overbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}^{v} + \sigma_w \tag{4.7}$$

$$\overbrace{\begin{bmatrix} u \\ y \end{bmatrix}}^{z} = \begin{bmatrix} C_1' \\ C_2' \end{bmatrix} x^k + \begin{bmatrix} D_{11}' & D_{12}' \\ D_{21}' & D_{22}' \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \sigma_v \tag{4.8}$$

whereas $v \in \Re^{(m+l)\times 1}, z \in \Re^{(m+l)\times 1}$. and σ_w, σ_v are the noise characteristic for the closed loop case. x_k represents the state of overall closed loop system, consisting of plant and controller states within some similarity transformation. If the plant and the controller states are known then the system matrices can be calculated as.

$$\begin{pmatrix} X_{i+1} \\ Z_{i|i} \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} X_i \\ V_{i|i} \end{pmatrix} + \begin{pmatrix} \sigma_w \\ \sigma_v \end{pmatrix} \tag{4.9}$$

Here X_i is the state vector sequence (refer Eq. 3.47) Also $Z_{i|i}$ *and* $V_{i|i}$ are the i^{th} row of the block Hankel matrix which can obtained using z_k *and* v_k respectively (refer Eq. 3.41). Comparing Eqs. 4.7 - 4.8 with Eq. 4.5 -. 4.6 we see that closed loop transfer function can be represented in term of plant and controller matrices within some similarity transformation between the states as

$$A' = T \begin{bmatrix} A_p - B_p D_c C_p & B_p C_c \\ -B_c C_p & A_c \end{bmatrix} T^{-1} \quad B' = T \begin{bmatrix} B_p D_c & B_p \\ B_c & 0 \end{bmatrix}$$
$$C' = \begin{bmatrix} -D_c C_p & C_c \\ C_p & 0 \end{bmatrix} T^{-1} \quad D' = \begin{bmatrix} D_c & I \\ 0 & 0 \end{bmatrix} \tag{4.10}$$

Also the noise characteristics for the closed loop case is the function of actual process and measurement noise. The new noise relationship can be represented as

$$\text{with} \quad E\left[\begin{pmatrix} \sigma_w \\ \sigma_v \end{pmatrix} \begin{pmatrix} \sigma_w^T \; \sigma_v^T \end{pmatrix}\right] = \begin{pmatrix} Q_s' & S_S' \\ (S_s')^T & R_s' \end{pmatrix} \delta_{kl} \geq 0 \quad \text{whereas} \tag{4.11}$$

$$\begin{aligned} Q_s' &= \begin{bmatrix} I & -B_p D_c \\ 0 & -B_c \end{bmatrix} \begin{pmatrix} Q_s & S_s \\ S_s^T & R_s \end{pmatrix} \begin{bmatrix} I & -B_p D_c \\ 0 & -B_c \end{bmatrix}^T \\ S_s' &= \begin{bmatrix} I & -B_p D_c \\ 0 & -B_c \end{bmatrix} \begin{pmatrix} Q_s & S_s \\ S_s^T & R_s \end{pmatrix} \begin{bmatrix} 0 & I \\ -D_c & 0 \end{bmatrix}^T \\ R_s' &= \begin{bmatrix} 0 & I \\ -D_c & 0 \end{bmatrix} \begin{pmatrix} Q_s & S_s \\ S_s^T & R_s \end{pmatrix} \begin{bmatrix} 0 & I \\ -D_c & 0 \end{bmatrix}^T \end{aligned} \tag{4.12}$$

We observe that actual plant noise characteristic can be obtained from any of the three Eq. 4.12. This implies that we have given too much degree to freedom in the estimation of close loop noise characteristics.

4.1.1 Extraction of Plant transfer function from the Overall transfer function

We know that the system matrices can be calculated, in a least squares manner if we know the inputs, outputs and the states of the system. System states can be calculated (as described in Sec 3.4.2) just from the knowledge of process inputs and outputs. But in the closed loop case, the calculated state consists of plant states and controller states. In present section we show that the least square calculation between the combined states (plant state and the controller state) along with the plant input and output will result in the identification of plant transfer function. Since this least squares solution results in the identification of just the plant state we will have to perform a minimal realization such that the unobservable (controller) states are eliminated form the identified plant transfer function.

Considering a system with an input-output relationship given as

$$\begin{pmatrix} x_{i+1} \\ y_i \end{pmatrix} = \mathcal{G} \begin{pmatrix} x_i \\ u_i \end{pmatrix} \quad \text{where} \quad \mathcal{G} = \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array} \right] \tag{4.13}$$

We define a system $\mathcal{G}^*$ as the inverse of $\mathcal{G}$, by retaining the same states and exchanging the role of inputs and outputs. If $\mathcal{D}$ is invertible the inverse system exists and can be written as follows:

$$\begin{pmatrix} x_{i+1} \\ u_i \end{pmatrix} = \mathcal{G}^* \begin{pmatrix} x_i \\ y_i \end{pmatrix} \quad \text{where} \quad \mathcal{G}^* = \left[\begin{array}{c|c} \mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C} & \mathcal{B}\mathcal{D}^{-1} \\ \hline -\mathcal{D}\mathcal{C}^{-1} & \mathcal{D}^{-1} \end{array} \right] \tag{4.14}$$

For closed loop identification the external signal can be injected either at the process input or at the setpoint. The overall state space transfer function will be different for each of the two cases. But the least square solution for each case should result in the identification of process transfer function. Considering the case when the signal is injected at the process input ($V_{i|i}^{(1)} = 0$). Input-output-state equation in matrix form can be written, after the inversion implied in (Eq. 4.14).

$$\begin{pmatrix} X_{i+1} \\ V_{i|i}^{(2)} \end{pmatrix} = \begin{pmatrix} A' - B_2' D_{12}'^{-1} C_1' & B_2' D_{12}'^{-1} \\ -D_{12}'^{-1} C_1' & D_{12}'^{-1} \end{pmatrix} \begin{pmatrix} X_i \\ Z_{i|i}^{(1)} \end{pmatrix} \tag{4.15}$$

$$\begin{pmatrix} X_{i+1} \\ V_{i|i}^{(2)} \end{pmatrix} = \begin{pmatrix} A' - B_2' D_{22}'^{-1} C_2' & B_2' D_{22}'^{-1} \\ -D_{22}'^{-1} C_2' & D_{22}'^{-1} \end{pmatrix} \begin{pmatrix} X_i \\ Z_{i|i}^{(2)} \end{pmatrix} \tag{4.16}$$

where $V_{i|i}^{(1)}, (\in \Re^{l \times j})$ is the first l rows of $V_{i|i}$ and $V_{i|i}^{(2)}, (\in \Re^{m \times j})$ represent the last m rows of $U_{i|i}$. Also $Z_{i|i}^{(1)}, (\in \Re^{m \times j})$ and $Z_{i|i}^{(2)}, (\in \Re^{l \times j})$ represent the first m and last l

rows of $Z_{i|i}$. Using Eq. 4.15 and Eq. 4.16 we can eliminate $V_{i|i}$. Hence a relationship between $Z^{(1)}_{i|i}$ and $Z^{(2)}_{i|i}$ can be obtained.

$$\begin{aligned} V^{(2)}_{i|i} &= -D'^{-1}_{12}C'_1X_i + D'^{-1}_{12}Z^{(1)}_{i|i} = -D'^{-1}_{22}C'_2X_i + D'^{-1}_{22}Z^{(2)}_{i|i} \\ Z^{(2)}_{i|i} &= (C'_2 - D'_{22}D'^{-1}_{21}C'_1)X_i + D'_{22}D'^{-1}_{21}Z^{(1)}_{i|i} \end{aligned} \tag{4.17}$$

Hence the state space representation for signal injected as process input with $Z^{(1)}_{i|i}$ as input , $Z^{(2)}_{i|i}$ treated as the output and X_i as the combined state vector can be represented in matrix form using Eq. 4.15 and 4.17 as.

$$\begin{pmatrix} X_{i+1} \\ Z^{(2)}_{i|i} \end{pmatrix} = \begin{pmatrix} A' - B'_2D'^{-1}_{12}C'_1 & B'D'^{-1}_{12} \\ C'_2 - D'_{22}D'^{-1}_{12}C'_1 & D'_{22}D'^{-1}_{12} \end{pmatrix} \begin{pmatrix} X_i \\ Z^{(1)}_{i|i} \end{pmatrix} \tag{4.18}$$

Similarly for signal injected at the setpoint we can derive the similar expression.

$$\begin{pmatrix} X_{i+1} \\ Z^{(2)}_{i|i} \end{pmatrix} = \begin{pmatrix} A' - B'_1D'^{-1}_{11}C'_1 & B'_1D'^{-1}_{11} \\ C'_2 - D'_{21}D'^{-1}_{11}C'_1 & D'_{21}D'^{-1}_{11} \end{pmatrix} \begin{pmatrix} X_i \\ Z^{(1)}_{i|i} \end{pmatrix} \tag{4.19}$$

It is proved in Appendix B that a least squares solution followed by a minimal realization results in the identification of process transfer function, modulo a transformation between the states.

$$\left[\begin{array}{c|c} A' - B'_2D'^{-1}_{12}C'_1 & B'D'^{-1}_{12} \\ \hline C'_2 - D'_{22}D'^{-1}_{12}C'_1 & D'_{22}D'^{-1}_{12} \end{array}\right] \iff \left[\begin{array}{c|c} \mathcal{T}^{-1}A_p\mathcal{T} & \mathcal{T}^{-1}B_p \\ \hline C_p\mathcal{T} & D \end{array}\right] \tag{4.20}$$

Concerning the step of Minimal Realization: It has been shown in the Appendix B that the process transfer function estimated by Eq. 4.18 or Eq. 4.19 will have some unobservable and uncontrollable parts in it. For the case when the input-output data is corrupted with noise this will result in the introduction of some uncanceled poles and zeros. These unwanted poles and zeroes should be cancelled from the obtained transfer function. The decision about the tolerance limit, within which the poles-zeros are to be cancelled, is an engineering choice depending on the specific application. Special care should be taken in the cancelation of unstable poles and zeros because the obtained transfer function is particularly sensitive to them. For the purpose of this thesis an inbuilt m-file (*minreal*) of Matlab was used and the tolerance limit was supplied from the knowledge of the estimated poles and zeros.

4.1.2 Extraction of Controller transfer function from the Overall transfer function

It has been proved in the appendix A that for $D_p = 0$ the controller output and input can be represented in terms of external signal and the overall state as

$$\begin{bmatrix} x_p^{k+1} \\ x_c^{k+1} \end{bmatrix} = \begin{bmatrix} A_p - B_pD_cC_p & B_pC_c \\ -B_cC_p & A_c \end{bmatrix}\begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} B_pD_c & B_p \\ B_c & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} I & -B_pD_c \\ 0 & -B_c \end{bmatrix}\begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{4.21}$$

$$\begin{bmatrix} y_c \\ u_c \end{bmatrix} = \begin{bmatrix} u - u_2 \\ u_1 - y \end{bmatrix} = \begin{bmatrix} -D_cC_p & C_c \\ -C_p & 0 \end{bmatrix}\begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} D_c & 0 \\ I & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & I \\ 0 & -D_c \end{bmatrix}\begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{4.22}$$

Moreover we can write an overall state space model between the external signal and the augmented controller output (consisting of controller input and output) as

$$x'^{k+1} = \mathcal{A}'x'^k + [\mathcal{B}_1' \ \mathcal{B}_2']\overbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}^{v'} + \sigma_w \tag{4.23}$$

$$\overbrace{\begin{bmatrix} y_c \\ u_c \end{bmatrix}}^{z'} = \begin{bmatrix} \mathcal{C}_1' \\ \mathcal{C}_2' \end{bmatrix} x'^k + \begin{bmatrix} \mathcal{D}_{11}' & \mathcal{D}_{12}' \\ \mathcal{D}_{21}' & \mathcal{D}_{22}' \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \sigma_v \tag{4.24}$$

Comparing Equation Eq.4.21 - 4.22 with Eq.4.23 -4.24 we see that closed loop transfer function can be represented in term of plant and controller matrices with some similarity transformation as

$$\mathcal{A}' = T \begin{bmatrix} A_p - B_p D_c C_p & B_p C_c \\ -B_c C_p & A_c \end{bmatrix} T^{-1} \quad \mathcal{B}' = T \begin{bmatrix} B_p D_c & B_p \\ B_c & 0 \end{bmatrix}$$
$$\mathcal{C}' = \begin{bmatrix} -D_c C_p & C_c \\ -C_p & 0 \end{bmatrix} T^{-1} \quad \mathcal{D}' = \begin{bmatrix} D_c & 0 \\ I & 0 \end{bmatrix} \tag{4.25}$$

But comparing Eq. 4.10 and Eq. 4.25 we see that matrices A', B', C', D' can be algerbrically manipulated to get $\mathcal{A}', \mathcal{B}', \mathcal{C}', \mathcal{D}'$ and vice versa

$$\mathcal{A}' = A', \quad \mathcal{B}' = B', \quad \mathcal{C}' = \mathcal{I}C', \quad \mathcal{D}' = \mathcal{J} + \mathcal{I}D' \tag{4.26}$$

$$\text{whereas} \quad \mathcal{I} = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \quad \mathcal{J} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \quad \text{hence we have} \tag{4.27}$$

$$\begin{gathered} \mathcal{C}_1' = C_1', \quad \mathcal{C}_2' = -C_2', \quad \mathcal{D}_{11}' = D_{11}' \\ \mathcal{D}_{12}' = -I + D_{12}', \quad \mathcal{D}_{21}' = I - D_{21}', \quad \mathcal{D}_{22}' = -D_{22}' \end{gathered} \tag{4.28}$$

Controller transfer function can be derived in the similar fashion as the plant transfer function was obtained. For the case of the controller the combined output consist of controller output, $y_c = u - u_2$, and controller input, $u_c = u_1 - y$. For the case when the signal is injected at process input we can write the input-state-output equation in matrix form as follows.

$$\begin{pmatrix} X_{i+1}' \\ \mathcal{V}_{i|i}^{(2)} \end{pmatrix} = \begin{pmatrix} \mathcal{A}' - \mathcal{B}_2' \mathcal{D}_{12}'^{-1} \mathcal{C}_1' & \mathcal{B}_2' \mathcal{D}_{12}'^{-1} \\ -\mathcal{D}_{12}'^{-1} \mathcal{C}_1' & \mathcal{D}_{12}'^{-1} \end{pmatrix} \begin{pmatrix} X_i' \\ \mathcal{Z}_{i|i}^{(1)} \end{pmatrix} \tag{4.29}$$

$$\begin{pmatrix} X_{i+1}' \\ \mathcal{V}_{i|i}^{(2)} \end{pmatrix} = \begin{pmatrix} \mathcal{A}' - \mathcal{B}_2' \mathcal{D}_{22}'^{-1} \mathcal{C}_2' & \mathcal{B}_2' \mathcal{D}_{22}'^{-1} \\ -\mathcal{D}_{22}'^{-1} \mathcal{C}_2' & \mathcal{D}_{22}'^{-1} \end{pmatrix} \begin{pmatrix} X_i' \\ \mathcal{Z}_{i|i}^{(2)} \end{pmatrix} \tag{4.30}$$

Here X_i' is the state vector sequence. Also $\mathcal{Z}_{i|i}$ *and* $\mathcal{V}_{i|i}$ are the i^{th} row of the block Hankel matrix which can obtained using z_k' *and* v_k'. Eliminating $\mathcal{V}_{i|i}^{(2)}$ from the above equation we get.

$$\begin{pmatrix} X_{i+1}' \\ \mathcal{Z}_{i|i}^{(1)} \end{pmatrix} = \begin{pmatrix} \mathcal{A}' - \mathcal{B}_2'\mathcal{D}_{22}'^{-1}\mathcal{C}_2' & \mathcal{B}_2'\mathcal{D}_{22}'^{-1} \\ \mathcal{C}_1' - \mathcal{D}_{12}'\mathcal{D}_{22}'^{-1}\mathcal{C}_2' & \mathcal{D}_{12}'\mathcal{D}_{22}'^{-1} \end{pmatrix} \begin{pmatrix} X_i' \\ \mathcal{Z}_{i|i}^{(2)} \end{pmatrix} \tag{4.31}$$

Substituting the value of $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ from Eq 4.26 - 4.28 in terms of A', B', C', D' we get

$$\begin{pmatrix} X_{i+1}' \\ \mathcal{Z}_{i|i}^{(1)} \end{pmatrix} = \begin{pmatrix} A' - B_2'D_{22}'^{-1}C_2' & -B_2'D_{22}'^{-1} \\ C_1' - (D_{12}' - I)D_{22}'^{-1}C_2' & -(D_{12}' - I)D_{22}'^{-1} \end{pmatrix} \begin{pmatrix} X_i' \\ \mathcal{Z}_{i|i}^{(2)} \end{pmatrix} \tag{4.32}$$

Similarly for the case of external signal injected at setpoint we have.

$$\begin{pmatrix} X_{i+1}' \\ \mathcal{Z}_{i|i}^{(1)} \end{pmatrix} = \begin{pmatrix} \mathcal{A}' - \mathcal{B}_1'\mathcal{D}_{21}'^{-1}\mathcal{C}_2' & \mathcal{B}_1'\mathcal{D}_{21}'^{-1} \\ \mathcal{C}_1' - \mathcal{D}_{11}'\mathcal{D}_{21}'^{-1}\mathcal{C}_2' & \mathcal{D}_{11}'\mathcal{D}_{21}'^{-1} \end{pmatrix} \begin{pmatrix} X_i' \\ \mathcal{Z}_{i|i}^{(2)} \end{pmatrix} \tag{4.33}$$

$$\begin{pmatrix} X_{i+1}' \\ \mathcal{Z}_{i|i}^{(1)} \end{pmatrix} = \begin{pmatrix} A' + B_1'(I - D_{21}')^{-1}C_2' & B_1'(I - D_{21}')^{-1} \\ C_1' + D_{11}'(I - D_{21}')^{-1}C_2' & D_{11}'(I - D_{21}')^{-1} \end{pmatrix} \begin{pmatrix} X_i' \\ \mathcal{Z}_{i|i}^{(2)} \end{pmatrix} \tag{4.34}$$

The interesting point to be noted here is that the controller transfer function can not be evaluated for the case when $D_p = 0$ and the signal is injected at the process input because then the matrix $\mathcal{D}_{22}'$ in Eq. 4.31 to be inverted in that case is singular. For the case when $D_p = 0$ and the signal is injected at setpoint the matrix equation reduces to Eq. 4.35.It has been proved in appendix C that least square solution followed by minimal realization results in the identification of controller transfer function.

$$\begin{bmatrix} A' + B_1'(I - D_{21}')^{-1}C_2' & B_1'(I - D_{21}')^{-1} \\ C_1' + D_{11}'(I - D_{21}')^{-1}C_2' & D_{11}'(I - D_{21}')^{-1} \end{bmatrix} \iff \left[\begin{array}{c|c} \mathcal{T}^{-1}A_c\mathcal{T} & \mathcal{T}^{-1}B_c \\ \hline C_c\mathcal{T} & D_c \end{array}\right] \tag{4.35}$$

A Note on the derivation: A closer look at Sec 4.1.1 and 4.1.2 reveals that the plant and the controller transfer function can be obtained in two different ways from the closed loop data. Firstly, evaluating the overall closed loop transfer function and then reducing it to the transfer function of interest. Secondly, posing the problem in a least square fashion such that we get the plant or controller transfer function directly. Most of the derivation presented in Sec. 4.1.1 and 4.1.2 has been presented to facilitate the understanding of second method. Details regarding both the methods are presented in the next section where we identify each of them as separate algorithms. It is also interesting to note that they have there own advantages and disadvantages, which are compared in Sec 4.2.

4.2 Closed Loop Subspace Identification Algorithms

In Sec. 3.4.2 details were presented regarding the open loop identification for deterministic case. The deterministic algorithm has to be modified a bit to handle the deterministic-stochastic case. Evaluation of state sequence for combined case (with the technique mentioned in Sec. 3.4.2) will result in a biased state sequence because of the presence of the stochastic component. Details regarding the combined case can be found in (Van Overschee, 1995). In this section we present two algorithms for closed loop deterministic-stochastic identification.

Algorithm 1 uses the unbiased estimate of the state sequence (*the combined deterministic-stochastic algorithm for open loop case*) to evaluate the overall closed loop transfer function. Appropriate matrix manipulations have to be done to obtain the plant and controller transfer function separately from the overall closed loop transfer function. Sometime this approach may result in the identification of an unstable transfer function even though the original system is stable. Moreover the Algorithm 1 is quite complex particularly in the evaluation of system matrices B and D, hence it is computationally intensive.

Algorithm 2 is a simplified version of Algorithm 1. It overcomes both drawbacks of Algorithm 1 (*i.e Stability and computational load*) by using the deterministic algorithm (*Sec. 3.4.2*). Since Algorithm 2 uses the deterministic algorithm it intrinsically uses the biased estimate of state sequence and hence will be less accurate. But our simulation example shows that the results from Algorithm 2 converge to those of Algorithm 1 as we increase the number of block row. It was observed that for $i > 30$ both the algorithm gives the same results.

4.2.1 Algorithm 1 : Using Unbiased estimate of State Sequence

(refer to (Van Overschee and De Moor, 1994b) for more details)

1. Pose the closed loop identification problem as a joint input-output problem

$$z = \begin{bmatrix} u \\ y \end{bmatrix}, \quad v = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4.36}$$

2. Calculate the oblique and orthogonal projection

$$\Theta_i = Z_f/_{V_f} W_p, \quad L_i = Z_f/\begin{pmatrix} W_p \\ V_f \end{pmatrix} \quad L_{i+1} = Z_f^-/\begin{pmatrix} W_p^+ \\ V_f^- \end{pmatrix} \tag{4.37}$$

3. Calculate the SVD of the oblique projection

$$\Theta_i = (U_1 \ U_2)\begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}$$
$$= U_1 S_1 V_1^T \tag{4.38}$$

4. Determine Γ_i and Γ_{i-1} as

$$\Gamma_i = U_1 S_1^{1/2} \ , \quad \Gamma_{i-1} = \underline{\Gamma_i} \tag{4.39}$$

5. Solve the linear set of equations

$$\begin{pmatrix} \Gamma_{i-1}^\dagger L_{i+1} \\ Z_{i|i} \end{pmatrix} = \begin{pmatrix} A' & \mathcal{K}_{11} \\ C' & \mathcal{K}_{12} \end{pmatrix}\begin{pmatrix} \Gamma_i^\dagger L_i \\ V_f \end{pmatrix} + \begin{pmatrix} \sigma_w \\ \sigma_v \end{pmatrix} \tag{4.40}$$

6. Solve for B' and D' from the knowledge of $A', C', \mathcal{K}_{11}, \mathcal{K}_{12}$ by minimizing the optimization criterion. (Van Overschee and De Moor, 1994c)

7. Determine as Q', S', R' from the residual as

$$E\left[\begin{pmatrix} \sigma_w \\ \sigma_v \end{pmatrix} \begin{pmatrix} \sigma_w & \sigma_v \end{pmatrix}\right] = \begin{pmatrix} Q'_s & S'_S \\ (S'_s)^t & R'_s \end{pmatrix} \delta_{kl} \geq 0 \tag{4.41}$$

8. Determine the Plant transfer function as

$$G_p = \begin{cases} \left[\begin{array}{c|c} A' - B'_2 D'_{12}{}^{-1} C'_1 & B'_2 D'_{12}{}^{-1} \\ \hline C'_2 - D'_{22} D'_{12}{}^{-1} C'_1 & D'_{22} D'_{12}{}^{-1} \end{array}\right] & \text{Signal injected at process input} \\ \left[\begin{array}{c|c} A' - B'_1 D'_{11}{}^{-1} C'_1 & B'_1 D'_{11}{}^{-1} \\ \hline C'_2 - D'_{21} D'_{11}{}^{-1} C'_1 & D'_{21} D'_{11}{}^{-1} \end{array}\right] & \text{Signal injected at Setpoint} \end{cases} \tag{4.42}$$

9. Determine the Controller transfer function as

$$G_c = \begin{cases} \left[\begin{array}{c|c} A' - B'_2 D'_{22}{}^{-1} C'_2 & -B'_2 D'_{22}{}^{-1} \\ \hline C'_1 - (D'_{12} - I) D'_{22}{}^{-1} C'_2 & -(D'_{12} - I) D'_{22}{}^{-1} \end{array}\right] & \text{Signal injected at process input} \\ \left[\begin{array}{c|c} A' + B'_1 (I - D'_{21})^{-1} C'_2 & B'_1 (I - D'_{21})^{-1} \\ \hline C'_1 + D'_{11} (I - D'_{21})^{-1} C'_2 & D'_{11} (I - D'_{21})^{-1} \end{array}\right] & \text{Signal injected at Setpoint} \end{cases} \tag{4.4}$$

10. Determine the Stochastic part of the open loop plant as

$$\begin{pmatrix} Q_s & S_s \\ (S_s)^t & R_s \end{pmatrix} = \begin{bmatrix} I & -B_p D_c \\ 0 & -B_c \end{bmatrix}^{-1} Q'_s \begin{bmatrix} I & -B_p D_c \\ 0 & -B_c \end{bmatrix}^{-T} \tag{4.44}$$

4.2.2 Algorithm 2: Using Biased Estimate of State Sequence

1. Same as steps 1-4 of Algorithm 1

2. Calculate overall state sequence as

$$X_i = \Gamma_i{}^{\dagger} \Theta_i \ , \ \ X_{i+1} = \Gamma_{i-1}{}^{\dagger} \Theta_{i-1} \tag{4.45}$$

3. Calculate the plant transfer function by solving the least square problem.

$$\begin{pmatrix} X_{i+1} \\ Z_{i|i}^{(2)} \end{pmatrix} = \begin{pmatrix} A_p & B_p \\ C_p & D_p \end{pmatrix} \begin{pmatrix} X_i \\ Z_{i|i}^{(1)} \end{pmatrix} \tag{4.46}$$

   The identified transfer function will have some uncanceled poles and zeros. Remove them by some minimal realization step. Refer to Sec. 4.1.1 for more details.

4. Calculate the controller transfer function by solving the least square problem followed by a minimal realization step.

$$\begin{pmatrix} X_{i+1} \\ \mathcal{Z}_{i|i}^{(1)} - \mathcal{V}_{i|i}^{(2)} \end{pmatrix} = \begin{pmatrix} A_c & B_c \\ C_c & D_c \end{pmatrix} \begin{pmatrix} X_i \\ \mathcal{V}_{i|i}^{(2)} - \mathcal{Z}_{i|i}^{(2)} \end{pmatrix} \tag{4.47}$$

Implementation Remark: It turns out that both the algorithms can be implemented in numerically efficient way by making extensive use of QR decomposition. It has been shown in (Van Overschee and De Moor, 1994b) that only R factor of a QR decomposition is required to calculated the matrix projection and the state sequences. The implementation of the algorithms has not been not discussed here. It is a very important aspect in application of both the algorithms and the interested reader may refer to (Van Overschee and De Moor, 1994b) for more details.

4.2.3 Technique to guarantee the Stability of the Algorithm 2

A general state space equation can be written as

$$\begin{aligned} X_{l+1} &= AX_l + Bu_l \\ X_{l+2} &= A^2X_l + [AB \quad B] \begin{bmatrix} u_l \\ u_{l+1} \end{bmatrix} \\ \vdots \quad & \quad \vdots \\ X_{l+i} &= A^iX_l + [A^{i-1}B \quad A^{i-2}B \ \cdots \ B] \begin{bmatrix} u_l \\ \vdots \\ u_{l+i-1} \end{bmatrix} \end{aligned} \tag{4.48}$$

For the value of l from $0 \cdots (j-1)$ we can write j copies of the equation and combine them as follows

$$\begin{aligned}[X_i\ X_{i+1}\ \cdots\ X_{i+j-1}] &= A^i[X_0\ X_1\ \cdots\ X_{j-1}] \\ &+[A^{n-1}B \quad A^{n-2}B\ \cdots\ B]\begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{bmatrix}\end{aligned} \tag{4.49}$$

$$X_f = A^i X_p + \Delta^i U_p \tag{4.50}$$

whereas Defining the reversed extended controllability matrix as

$$\begin{aligned} \Delta^i &= [A^{i-1}B \quad A^{i-2}B\ \cdots\ B] \\ \Delta^i_R &= [A^{i-2}B \quad A^{i-3}B\ \cdots\ B] \\ \Delta^i_L &= [A^{i-1}B \quad A^{i-2}B\ \cdots\ AB] \end{aligned} \tag{4.51}$$

Projecting the state propagation equation on the rows perpendicular to the past output $U_p$we can calculate the initial state sequence as

$$\begin{aligned} X_f\Pi_{U_p^\perp} &= A^n X_p^d \Pi_{U_p^\perp} + \Delta^n \underbrace{U_p\Pi_{U_p^\perp}}_{=0} \\ A^n X_p &= X_f/U_{p^\perp}(\Pi_{U_p^\perp})^\dagger \end{aligned} \tag{4.52}$$

Hence the reversed extended controllability matrix can be obtained as

$$\Delta^i = \left[X_f - X_f/U_{p^\perp}(\Pi_{U_p^\perp})^\dagger\right] U_p{}^\dagger \tag{4.53}$$

From the knowledge of the controllability matrix the system matrix can be obtained as

$$\begin{aligned} &A[A^{i-1}B \quad A^{i-2}B\ \cdots\ B] = [A^iB \quad A^{i-1}B\ \cdots\ AB] \\ &A\Delta^i = [A^iB \quad \Delta^i_L] \\ &A = [A^iB \quad \Delta^i_L]\Delta^{i\dagger} \end{aligned} \tag{4.54}$$

But if we estimate the value of A by introducing zeros in the first n_u column of the right hand side it has been proved in (Maciejowski, 1995) that the resultant matrix is always stable. is

$$\hat{A} = [0 \quad \Delta_L^n]\Delta^{n\dagger} \quad \text{is always stable} \tag{4.55}$$

4.3 Simulation

4.3.1 Example 1 : A Forth order SISO system with PI controller

We illustrate the effectiveness of the proposed algorithm by considering a single input single output system. The plant and the controller transfer functions are given as

$$G_p = \frac{1}{(\tau_1\tau_2 s^2 + \xi_1(\tau_1+\tau_2)s + 1)(\tau_3^2 s^2 + 2\xi_2\tau_3 s + 1)} \qquad G_c = K_c\frac{\tau_I s + 1}{\tau_I s} \tag{4.56}$$

for $\tau_1 = 30, \tau_2 = 40, \tau_3 = 50$ and $\xi_1 = .417, \xi_2 = 2$ the controller settings were found using Z-N tuning rule.($K_c = 2, \tau_I = 175$). The output of the plant is corrupted by a zero mean white noise sequence. To gain some insight about the strength of the algorithm for various white noise sequences, we define noise to signal ratio as follows with $y'(i)$ denotes the signal corresponding to $y(i)$ but with no noise. For this simulation noise to signal ratio of .35 was used.

$$\text{Percentage noise to signal ratio:} \quad \frac{\sqrt{\sum_{i=1}^{N} \| y(i) - y'(i) \|_2^2}}{\sqrt{\sum_{i=1}^{N} \| y'(i) \|_2^2}} \times 100$$

The figure 4.2(a) shows the open and closed loop step response of the system. The open loop settling time (800 sec) is reduced to 400 sec for the assumed controller tuning. [1] Identification was done for both the case where the signal was injected at the process input as well as at the setpoint. The design parameter for PRBS signal were selected such that the designed signal had sufficient power in the frequency

[1]Settling time is considered as the time taken to reach ± 0.05 of the steady state value

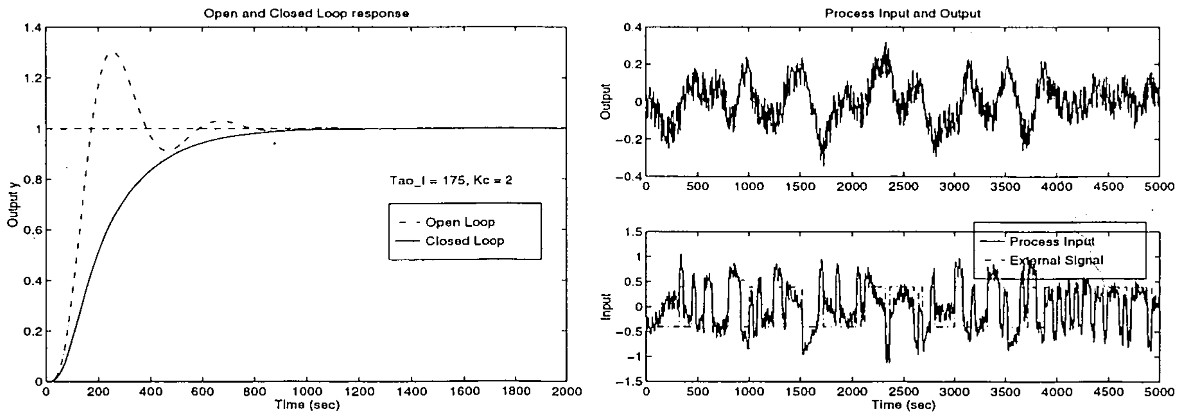


Figure 4.2: (a,left) Step response of open and the closed loop system (b,right) Process Output, Process Input and External Signal during identification experiment

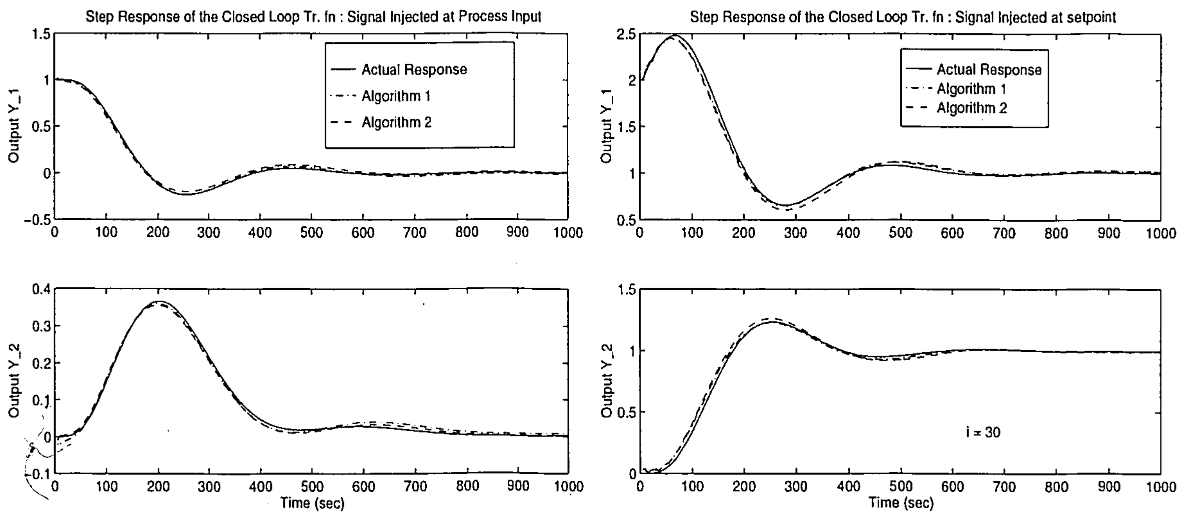


Figure 4.3: Response of the identified closed loop system for a unit step in the external signal. The external Signal is at the process input (a,left). The external signal is applied at the setpoint (b,right)

range of interest. (N=8, Tcl = 50). A sampling period of 5 sec was chosen. The process was simulated in Matlab using Simulink and the signal generator was used to generate the white noise sequence. 1000 samples were collected and both the algorithm were considered for the identification purpose. Order of the overall system was chosen from the plot of dominant singular of the oblique projection. Figure 4.2(b) shows the response of the system for the applied PRBS signal. It can be noticed form the figure that the signal to noise ratio is quite high.

Figure 4.3(a) shows step response of the identified closed loop system for the signal injected at process input. Number of block rows used for the identification purpose was 30. Both the algorithms predicts the step response of the closed loop system very well. This suggests that for the given noise to signal ratio and $i = 30$ the identified states of Algorithm 2 converges to the unbiased state estimate. This proves the fact that Algorithm 2 predict the actual system for relatively small values of block rows quite well. Figure 4.3(b) shows the step response of the identified process model for signal injected at setpoint. It indicates that the proposed identification algorithm captures all the important dynamics of the system.

Figure 4.4(a) compares the step response of the identified plant transfer function for various algorithms. The results are compared with direct identification to obtain some more insight. It can be seen that both the algorithms results is a good process model as compared to direct identification. Increasing the number of block rows results in the identification of more accurate model for all the methods. This again reiterates the fact the Algorithms 1 and 2 result in similar model for $i = 30$. Figure 4.4 (b) shows the frequency response of the identified model. It indicates that subspace algorithm does a reasonable job in identifying process transfer function for the desired frequency range. This could be due to the low input power spectrum in the higher frequency range.

Table 4.2 compares the poles [2] [3] of the identified transfer function for various values of block rows. It suggests that both the algorithm identifies the actual system

[2] Calculation of Plant Poles: The transfer function in continuous time was converted to state space form using *tf2ss* m-file of Matlab. The state space matrices in continuous time was converted to discrete time using *c2dm* using zero order hold

[3] Overall closed loop system matrices and hence the closed loop was calculated using Eq. 4.10

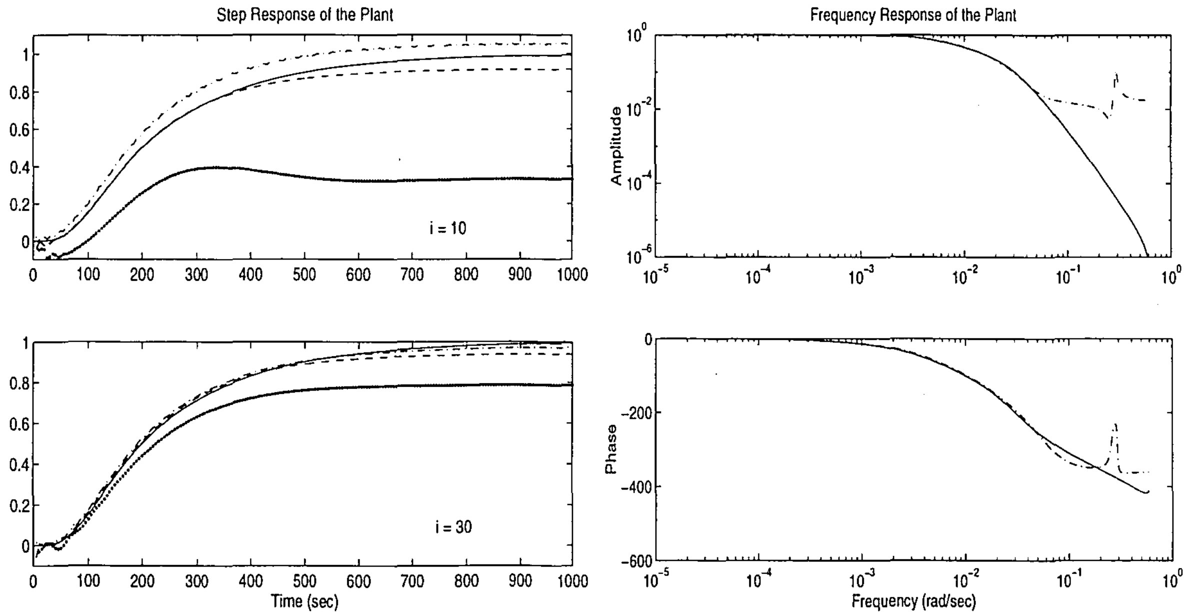


Figure 4.4: Response of the identified closed loop system for a unit step at the setpoint (a,left); Actual response (-,thin solid), Algorithm1 (– dashed), Algorithm2 (-.-.- dash dotted), Direct Identification (- thick solid). Frequency response of the identified transfer function (b,right) Actual response (-,thin solid), Identified Transfer function (-.-. dash dotted)

poles as the number of block rows is increased. It is interesting to note that both algorithms do a reasonable job of identifying the lightly damped poles (poles closer to unit circle). But there is some bias in identification of high frequency poles. This can also be seen from the frequency response of the identified plant transfer function.

Identified System	Actual	Algorithm: 1	
		i = 20	i = 100
Overall Closed	0.7017	0.1254	0.1024
Loop Poles	0.8546	0.9444	0.8744
	0.9651 + .074i	0.9558 + .0754i	0.9653 + .0749 i
	0.9651 - .074i	0.9558 - .0754i	0.9653 - .0749 i
	0.9686	0.9547	0.9585
Plant Poles	0.6885	0.1245	0.1003
	0.8966 + .0899 i	0.9151 + .0969i	0.9007 + .0813
	0.8966 - .0899 i	0.9151 - .0969i	0.9007 - .0813
	0.9736	0.9515	0.9726
Controller Poles	1.000	1.0193	1.0036

Table 4.1: Poles of the identified transfer function for Algorithm 1. The signal is injected at the setpoint.

Identified System	Actual	Algorithm: 1	
		i = 20	i = 100
Overall Closed	0.7017	0.1279	0.0690
Loop Poles	0.8546	.9109 + .0445 i	0.8731
	0.9651 + .074i	0.9643+ .0729i	0.9653 + .0749 i
	0.9651 - .074i	0.9643 -.0729i	0.9653 - .0749 i
	0.9686	0.9109 - 0.0445i	0.9584
Plant Poles	0.6885	0.1270	0.0666
	0.8966 + .0899 i	0.9100 + .0852i	0.9002 + .0816
	0.8966 - .0899 i	0.9100 + .0852i	0.9002 + .0816
	0.9736	0.9725	0.9727
Controller Poles	1.000	0.992	1.0017

Table 4.2: Poles of the identified transfer function for Algorithm 2. The signal is injected at the setpoint .

4.3.2 Example 2 : A 2nd order SISO system with PI controller :Guaranteed Stability

This example shows the application of the proposed modification in the algorithm such that a stable transfer function can be obtained from the closed loop identification. We consider here a discrete time second order system with having a state space representation given as:

$$
\begin{aligned}
x_{k+1} &= \begin{bmatrix} .97703 & -.00006 \\ .98847 & 0.99997 \end{bmatrix} x_k + \begin{bmatrix} .98847 \\ .49615 \end{bmatrix} u_k \\
y_k &= \begin{bmatrix} -.32342 & .09889 \end{bmatrix} x_k - 0.977 u_k + v_k
\end{aligned}
\tag{4.57}
$$

The poles of the assumed systems are at $0.98, 0.997$. This represents that low frequency dynamics are important for this system. The subspace identification method tends to compute unstable models for such system. A PI controller was tuned using the Z-N tuning rules. The system was excited by injecting a PRBS signal at the setpoint. 5000 data points were collected. The noise to signal ratio for the simulation was around 0.35. Plant matrices were identified using the proposed algorithm and the results are summarized in Table 4.3. It can be seen that both algorithms compute unstable model for $i = 10$. Results from the stability algorithm indicates that the unstable poles are pushed inside the unit circle to obtain a stable model. But the identification technique pushes all the system poles (stable as well as unstable) further away from the unit circle. Hence the technique should be used with extreme precaution because it might result in highly biased model. The proposed modification tends to place the actual system poles far inside the unit circle, resulting in greater bias for high frequency components. The example illustrates that a stable model can be obtained only at the cost of introducing bias in the identified model. It should be observed that both algorithms directly compute a stable model as the number of block rows are increased to 30. So the proposed stability technique should only be used as a last resort to compute a stable model.

Most of the results discussed here refer to SISO examples. The subspace algorithm reveals its full potential in dealing with MIMO cases. We have applied it

Number of Block Rows	Actual	Algorithm 1	Algorithm 2	Algo 2 with Modification
$i = 10$	0.997	1.00456	1.01062	0.415 + 0.2383i
	0.98	0.96212	0.91257	0.415 - 0.2383i
$i = 30$	0.997	0.98906+0.0068i	0.9986	
	0.98	0.98906-0.0068i	0.9774	

Table 4.3: Poles of the identified transfer function for signal injected at the setpoint. Noise to signal ratio was 0.35

successfully to the Amoco Fluid Catalytic Cracker Unit (FCCU). For the conciseness of this report this MIMO example is presented separately. The detailed model for the Amoco FCCU was developed by combined effort between Lehigh University and Amoco Corporation (McFarlane et al., 1993). This dynamic model captures the important nonlinearities, multi-variable interaction along with the constraints. The developed algorithms were successfully applied to this MIMO (4×4) case. and the results were compared with the Two Step Method, detailed in previous report (Jha et al., 1995). The subspace identification method proves to be more accurate.

4.4 Conclusions

- *Is was proved that plant and controller transfer function can be obtained separately from the overall closed loop transfer function by performing appropriate matrix manipulation. Subspace based state space algorithm was developed to identify process and controller transfer function for closed loop data.*

- *Simulation studies on a known linear system suggests that Algorithm 1, combined deterministic-stochastic approach, and Algorithm 2, the deterministic approach, gives similar results as the number of block rows in increased in the identification algorithm.*

- *The developed algorithm performs better than the classical identification technique like direct identification.*

- *Both the algorithms identifies the plant and controller transfer function accurately, from the closed loop data, even in the presence of unmeasured disturbances.*

- *Identified model, plant as well as controller, contains some unobservable and uncontrollable part, which has to be removed by performing a minimal realization step. The decision of the tolerance limit, within which the poles and zeros are to be cancelled, is an engineering judgment.*

- *The stability modification helps in the computation of stable model even for system with lightly damped poles. But the bias introduces by this modification is quite high and hence it should be used with caution.*

Chapter 5

Application To MIMO Fluid Catalytic Cracking Unit

The chapter presents the simulation results of closed loop identification algorithms on FCCU. Section 5.1 gives a brief introduction about the closed loop subspace algorithm which were discussed in the previous chapter. Section 5.2.1 describes the Fluid Catalytic Cracking Unit and indicates the manipulated and controller variable used for identification purpose. In Section 5.2.2 details regarding the controller settings, input signal design and signal injection in closed loop is discussed. Model parameterization for the Two Step and the Subspace identification method is described in Section 5.2.3. Comparative study of the model identified using Two step method and subspace identification is given in Section 5.2.4. The identified model are compared with the actual system in their step and frequency response. Section 5.3 summarizes the results which were obtained form this simulation study.

5.1 Introduction: Subspace Space Identification Algorithm

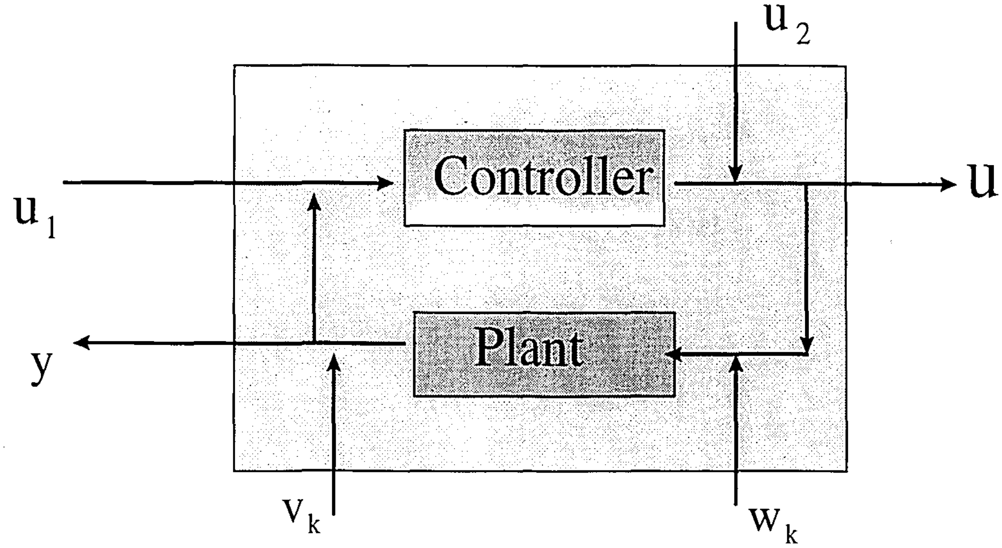
Subspace Identification identifies the state space model of a system directly from the the input-output data. They are the input-state-output generalization of the classical realization theory as developed in the sixties. Modern day subspace identification techniques like N4SID (Van Overschee and De Moor, 1994b), MOESP (Verhaegen, 1994) and CVA (Larimore, 1990) uses the powerful tools like Singular Value Decomposition (SVD) and QR factorization for effective numerical implementation. As described in Fig.5.2 subspace algorithm (N4SID) for open loop case calculates the input-output Hankel Matrix (Oblique Projection) by projecting the future output data Hankel matrix along the future input data onto the compound matrix of input-output data Hankel matrix. System order is calculated form the knowledge of dominant singular values of the oblique projection. The extended observability matrix and the state sequence can then be calculated from the system order and the SVD of the oblique projection. Once the system states are known the system matrices are calculated form the input-state-output data by solving the least squares problem.

For the state space description of plant and controller given by Eq 5.1 and 5.2 subspace algorithms are were developed (Sec 4.2) to identify plant and controller transfer functions separately.

$$\begin{aligned} \text{Plant:} \quad x_p(k+1) &= A_p x_p(k) + B_p u(k) + w_k \\ y(k) &= C_p x_p(k) + D_p u(k) + v_k \end{aligned} \tag{5.1}$$

$$\begin{aligned} \text{Controller} \quad x_c(k+1) &= A_c x_c(k) + B_c u_c(k) \\ y_c(k) &= C_c x_c(k) + D_c u_c(k) \end{aligned} \tag{5.2}$$

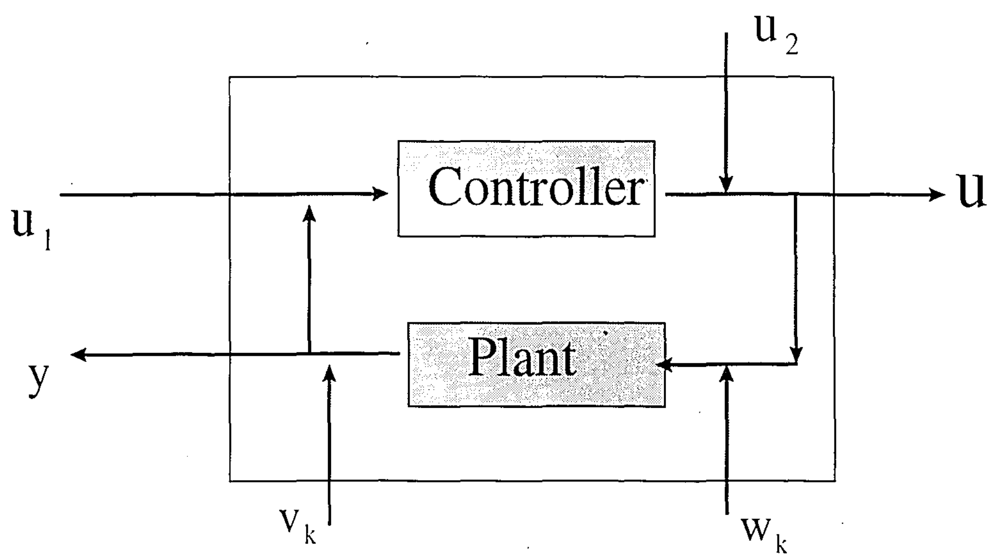
Open loop identification techniques are not directly applicable to closed loop data due to the correlation between input and unmeasured disturbances. Two algorithms



$$\begin{pmatrix} x_p^{k+1} \\ x_c^{k+1} \end{pmatrix} = A_{cl} \begin{pmatrix} x_p^{k} \\ x_c^{k} \end{pmatrix} + B_{cl} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y \\ u \end{pmatrix} = C_{cl} \begin{pmatrix} x_p^{k} \\ x_c^{k} \end{pmatrix} + D_{cl} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Figure 5.1: Joint input-output identification for closed loop system



$$\begin{pmatrix} x_p^{k+1} \\ x_c^{k+1} \end{pmatrix} = A_{cl} \begin{pmatrix} x_p^{k} \\ x_c^{k} \end{pmatrix} + B_{cl} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y \\ u \end{pmatrix} = C_{cl} \begin{pmatrix} x_p^{k} \\ x_c^{k} \end{pmatrix} + D_{cl} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Figure 5.1: Joint input-output identification for closed loop system

were developed to solve the closed loop problem by posing it as a joint input-output identification problem. Fig. 5.3 summarized the proposed closed loop identification algorithm. Algorithm 1 uses the unbiased estimate of the state sequence (the combined deterministic-stochastic algorithm for the open loop case) to evaluate the overall closed-loop transfer function from which the plant and the controller models could be identified separately by applying appropriate matrix manipulation.

Algorithm 2 is a simplified version of Algorithm 1. It uses the deterministic identification algorithm to come up with an estimate of state sequence for the joint input-output identification problem. The advantage of using deterministic technique over the combined deterministic-stochastic case is two fold. First, it leads to substantial reduction in the computational load and makes the algorithm more transparent. Second, the stability of the identified model can be guaranteed in a similar fashion as in the open-loop case. For the purpose of this chapter we have used Algorithm 1 for the identification of the process model using subspace identification. The step by step details of the Algorithm 1 has been detailed in Sec 4.2

5.2 Simulation Example :FCCU with PID controller (MIMO case)

5.2.1 Process Details: Fluid Catalytic Cracking Unit

Efficient operation of Fluid Catalytic Cracking Unit (FCCU) is vital for any refinery operation. FCCU processes the feed containing high boiling point component and cracks them into lighter streams which are blended with other streams from refinery to produce various grades of gasoline. A schematic flow diagram of Model IV FCCU is shown in figure 5.4. Model IV FCCU consists of two main section; the reactor, where the feed is mixed with hot regenerated catalyst to give rise to catalytic reactions, and the regenerator, where combustion reactions take place to burn the hydrocarbon deposited during cracking. A rigorous model for the Amoco

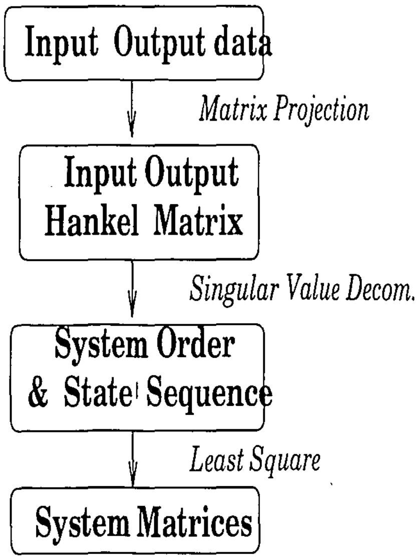


Figure 5.2: Subspace Identification Algorithm (N4SID) for Open loop data

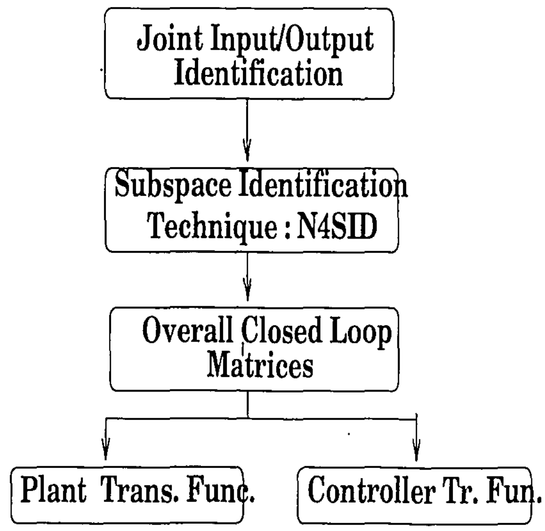


Figure 5.3: Closed Loop Subspace Identification Algorithm

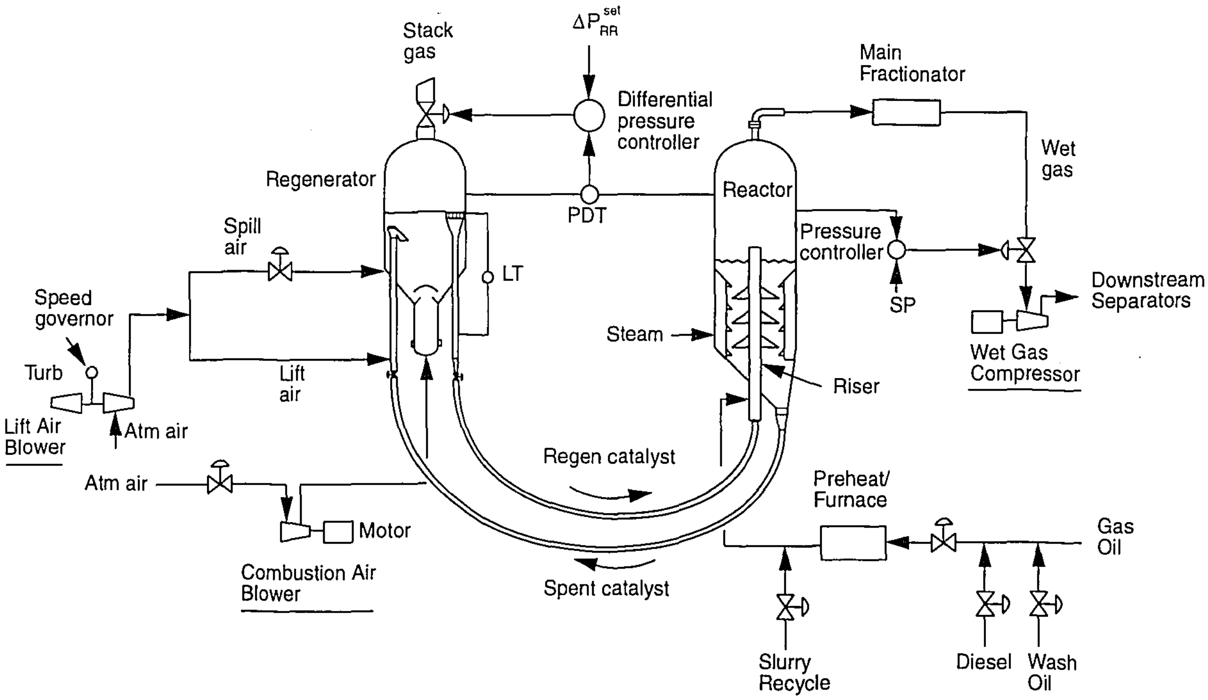


Figure 5.4: Model of Fluidized Catalytic Cracking Unit

FCCU was developed by combined effort of CPMC, Lehigh University and Amoco corporation (McFarlane et al., 1993). This dynamic model captures the important nonlinearities, multi-variable interaction along with equipment and operating constraints , arising from the economical, environmental and safety considerations. The input and outputs used for our simulation studies are detailed in Table. 5.1

Manipulated Variables	Feed flow rate (lb/s)	$F3_{set}$
	Slurry recycle flow rate (lb/s)	$F4_{set}$
	Liftair flow rate (lb/s)	$F9_{set}$
	Reactor-Reg. differential pressure (psi)	D_{pset}
Controlled Variables	Wet gas compression suction valve position	V_{11}
	Regenerator Bed temperature (F)	T_{reg}
	Stack gas carbon monoxide conc. (ppm)	CO_{sg}
	Reactor riser temperature (F)	T_r

Table 5.1: Manipulated and Controlled Variable for Model IV FCCU

5.2.2 System Identification

Controller Tuning (MIMO case): A digital version of the continuous time PI controller was used for the simulation purpose. Controller settings were determined from the Z-N tuning rules. The knowledge of ultimate gain K_u and ultimate frequency w_u was obtained from the open loop experimentation. Table 5.3 summarizes the results of ultimate gain, ultimate frequency and the steady state gain for the Amoco FCCU obtained from open loop experiments. A 4 x 4 system was chosen for MIMO identification purpose. Pairing was determined by analyzing the results obtained by NI, MRI and RGA analysis. The pairing which gave the positive NI index and the highest value of MRI index was chosen for closing the loop. Each loop was detuned by a factor between 2 and 5. Some fine tuning was done for the individual loop so as to get the desired response. Controller settings for the closed loop MIMO operation is detailed in Table 5.3

Excitation Signal (MIMO case): Special care has to be take for designing PRBS for MIMO case because it is difficult to predict when certain combination

I/O variable	**Input variables**					
	Feed			Slurry		
Output	K_u	w_u	k	K_u	w_u	k
T_r	-4	0.3	-0.231	1.1	.0025	2.41
T_{reg}	3.58	.0017	1.77	2.5	.003	8.62
$O2_{sg}$	-100	0.009	-.0124	-50	0.013	-0.423
CO_{sg}	0.4	0.01	26.8	0.4	0.015	69.48
V_{11}	180	0.2	0.017	250	0.2	0.048
Pairing	Feed- Co_{sg}			Slurry- T_{reg}		
K_c	0.04			0.3		
τ_I	400			1400		

Table 5.2: Ultimate Gain (K_u), Ultimate frequency (w_u), Steady State Gain (k) and Controller Settings (τ_I, K_c) for Amoco FCCU obtained from open loop experiments

I/O variable	**Input variables**					
	Liftair			Diff. Pressure		
Output	K_u	w_u	k	K_u	w_u	k
T_r	6.6	0.1	2.46	-0.25	0.3	-29.42
T_{reg}	2.75	0.002	2.78	-0.22	.0022	-37.32
$O2_{sg}$	-8.9	0.005	-0.072	1.053	0.01	1.99
CO_{sg}	0.04	0.006	11.84	-.0067	0.015	-393.7
V_{11}	180	0.05	0.03	-5.0	0.075	-.337
Pairing	Liftair- V_{11}			Diff Pressure- T_r		
K_c	12			-0.007		
τ_I	200			800		

Table 5.3: Ultimate Gain (K_u), Ultimate frequency (w_u), Steady State Gain (k) and Controller Settings (τ_I, K_c) for Amoco FCCU obtained from open loop experiments

of input might drive the process to an undesirable operating point. Independent PRBS design for each input or multiple delayed copies of a single PRBS can be used for MIMO identification. The independent PRBS design (used in this paper) should have negligible cross-correlation function $\Phi_{u_i u_j}(\tau)$ for each input pair for all τ less than the maximum settling time. T_{cl} (minimum switching time) and N (No. of shift registers.) are the two important design parameters for PRBS design.

To cover the desired frequency range a combination of two PRBS signal was chosen for each of the manipulated variable. For feed flow rate as a manipulated variable, a switching time of 100 sec with 6 shift registers was found to be a good choice for high frequency content, whereas switching time of 250 sec and 5 shift register covered the intermediate frequency range . The minimum length of PRBS test for the former case can be calculated as $(2^n - 1)T_{cl} = 6300sec$, whereas for the latter case it is 7750 sec. The test was conducted such that at least two repetitions of each input sequence occur. Hence the combined input,obtained by appending the one PRBS signal to the other, was conducted for the length 40,000 sec. A total of 8000 samples were generated for the sampling rate of 5 sec. Amplitude of excitation was chosen by trial and error for each of the input. The designed signal is summarized in Table 5.4

Manipulated	**PRBS 1**				**PRBS 2**			
Variables	n	Tcl	Amplitude	Samples	n	Tcl	Amplitude	Samples
Feed	6	100	0.4	4000	5	250	0.4	4000
Slurry	5	100	0.2	1500	6	250	0.1	6500
Liftair	7	100	0.3	5000	4	250	0.3	3000
Diff. pr.	4	100	0.03	1000	6	175	0.03	7000

Table 5.4: MIMO closed loop PRBS Input

Signal Injection in closed loop: The external signal can be injected at two different places in the closed loop at the setpoint and at the process input. A priori information about the closed loop sensitivity function $S(z)$ as well as the controller transfer function $C(z)$ is needed for deciding the location of signal injection. Hence the experiment design should truly be an iterative procedure. If the signal is introduced at setpoint it will be weighted by the function $S(z)C(z)$, whereas if it is introduced as a process input it will be modified by $S(z)$. Hence input to the process will be amplified or attenuated depending on the amplitude plot of $S(z)$ or $C(z)$. Bias during identification can be reduced by emphasizing those frequencies where $S(z)$ or $S(z)C(z)$ is low.

Controller settings play an important role in shaping the amplitude plots. If the amplitude of a controller transfer function is less than one for all frequencies then the signal injected at setpoint,r, will be attenuated as compared to the equivalent one at u_d. Whereas if it is greater than one it will be amplified. This type of behavior will play as important role in bias distribution. As a general rule of thumb the external signal should be injected at the location such that the weighing function ($S(z)C(z)$ *or* $S(z)$) is almost flat. This will give more control in shaping the bias distribution of the identified transfer function by changing the input spectrum. For the purpose of this paper the excitation signal was injected at process input, u_d, for the identification experiments. Form the preliminary knowledge of the sensitivity function and controller tuning it was found that the signal will be attenuated hence the input amplitude was increased to reduce the noise contribution in the signal.

5.2.3 Closed Loop Identification Methodology

Two Step Method : The process and noise model should be independently parameterized to get a consistent estimate of the process transfer function. Output Error (OE) [1] model satisfies this criterion and hence was used for the parameterization of the transfer function. In the first step of two step method high order OE models were used in the parameterization of closed loop sensitivity function. The estimate of closed loop sensitivity function from the first step was then used to calculate the noise free reconstructed signal. It can be seen from the Fig 5.6, that

[1]The output error model is given as

$$y(t) = \frac{b_1 + b_2q^{-1} + b_3q^{-2} + \cdots + b_{n_b}q^{-n_b+1}}{1 + f_1q^{-1} + f_2q^{-2} + \cdots + f_{n_f}q^{-n_f}}u(t - n_k) + e(t)$$

the reconstructed signal smoothenes out the effect of noise in the manipulated variable. Knowledge of the open loop SISO model orders were used in model parameter estimation for the second step. Model order used in the second step is detailed in Table 5.5

Input/Output variable	Input variables											
	Feed			Slurry Flow			Liftair Flow			Diff pr.		
Process output	n_b	n_f	n_k	n_b	n_f	n_k	n_b	n_f	n_k	n_b	n_f	n_k
Riser temp (Tr)	12	10	1	13	10	1	13	10	1	14	10	1
Regenerator temp (Treg)	5	4	1	5	3	1	5	3	1	5	3	1
Oxygen (O2sg)	9	7	1	9	6	1	9	6	1	13	7	1
Carbon mono. (COsg)	9	7	1	9	7	1	13	8	1	15	11	1
Valve (V11)	9	8	1	12	10	1	12	10	1	12	10	1

Table 5.5: Model order used in the 2nd step of Two step Method.

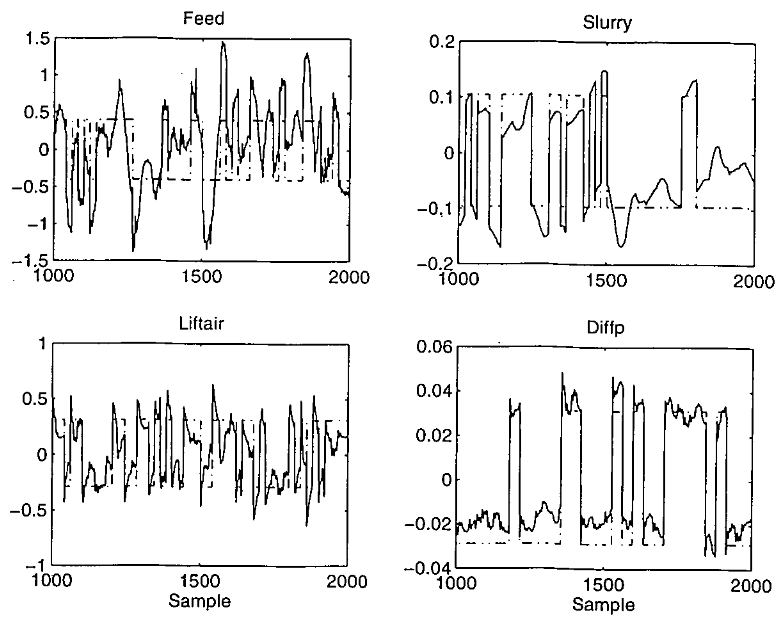


Figure 5.5: External Signal, u_d (dash-dotted) and Process Input, u(solid) for the Amoco FCCU

Subspace Identification : Algorithm detailed in Sec 4.2 was used to identify state space model from the input-output data. Model order was selected from the knowledge of the singular values of the oblique projection. A 15th order state space

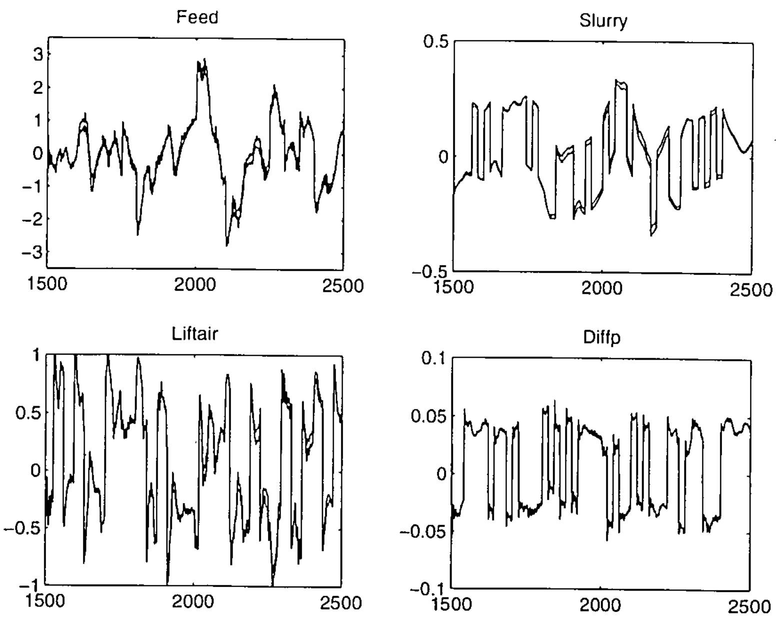


Figure 5.6: Process Input (solid), reconstructed signal (dash-dotted) from the first step of TS Method

model was identified for the 4x4 system. To get a rough idea of the achievable model quality, the coherence C was calculated for different combination of input and output signals using Eq. 5.3. It is a measure of the linear relationship between two signals. If C_{xy} is close to unity, it indicates that good linear model can be estimated. For C_{xy} close to zero, not much can be expected from the linear model. They should be used judiciously because in practice the spectral estimate is calculated from finite data length the and the variance of the resultant estimate could be quite large.

$$C_{xy}(\omega) = \frac{\Phi_{yx}(\omega)\Phi^{*}_{yx}(\omega)}{\Phi_{x}(\omega)\Phi_{y}(\omega)} \tag{5.3}$$

For MIMO system the sum of the coherence between all the inputs and any output should be equal to one. Fig 5.7 indicates that the relationship between Feed-Treg is not linear for lower frequency ranges and the identified steady state gain can be highly off the mark. Fig 5.8 indicates that relationship between Slurry-Treg could be nonlinear in nature because for most of the frequency range C_{xy} is quite small. (around 0.01). Hence the linear transfer function may not be able to predict the system behavior very well.

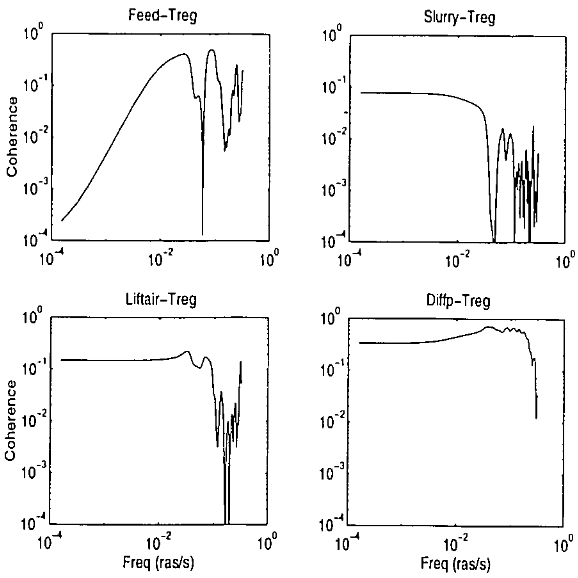


Figure 5.7: Coherence Spectrum of T_{reg} with respect to the manipulated variable of Amoco FCCU

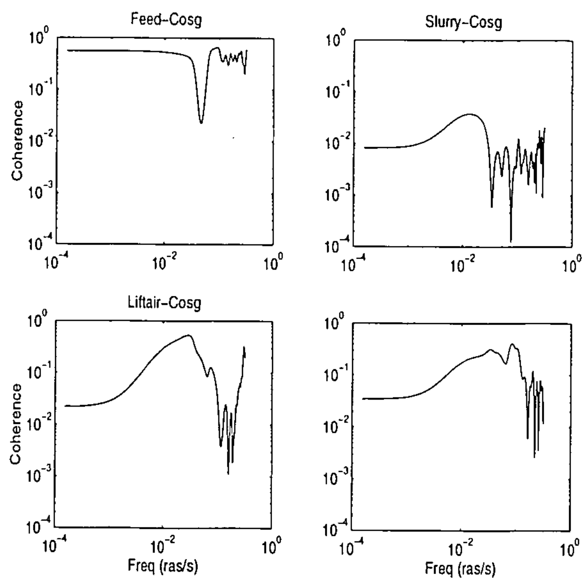


Figure 5.8: Coherence Spectrum of Co_{sg} with respect to the manipulated variable of Amoco FCCU

5.2.4 Identified Models

Fig 5.9 compares the actual and simulated output for both the identification methodology. It indicates that both the methods (TS method and Subspace method) does a reasonable job in predicting the system response from the given input data. But for the output T_{reg} and Co_{sg}, the prediction from subspace model is better than that of Two Step method. Table 5.7 summarizes the ultimate gain, ultimate frequency and the steady state gain of the identified model using subspace identification. Comparing details from Table 5.3 and 5.7 we notice that ultimate gain and ultimate frequency is correctly identified for almost all input-output pair. For some of the output Two Step method resulted in meaningless results. It resulted in identification of unstable model. This could be due to the nonlinear nature of the optimization procedure involved in the estimation of the model parameters.

Fig 5.10 - 5.11 compares the step response of the identified model with the actual system response . It indicates that subspace algorithm does a better job in predicting the step response than the Two Step method. The step responses of

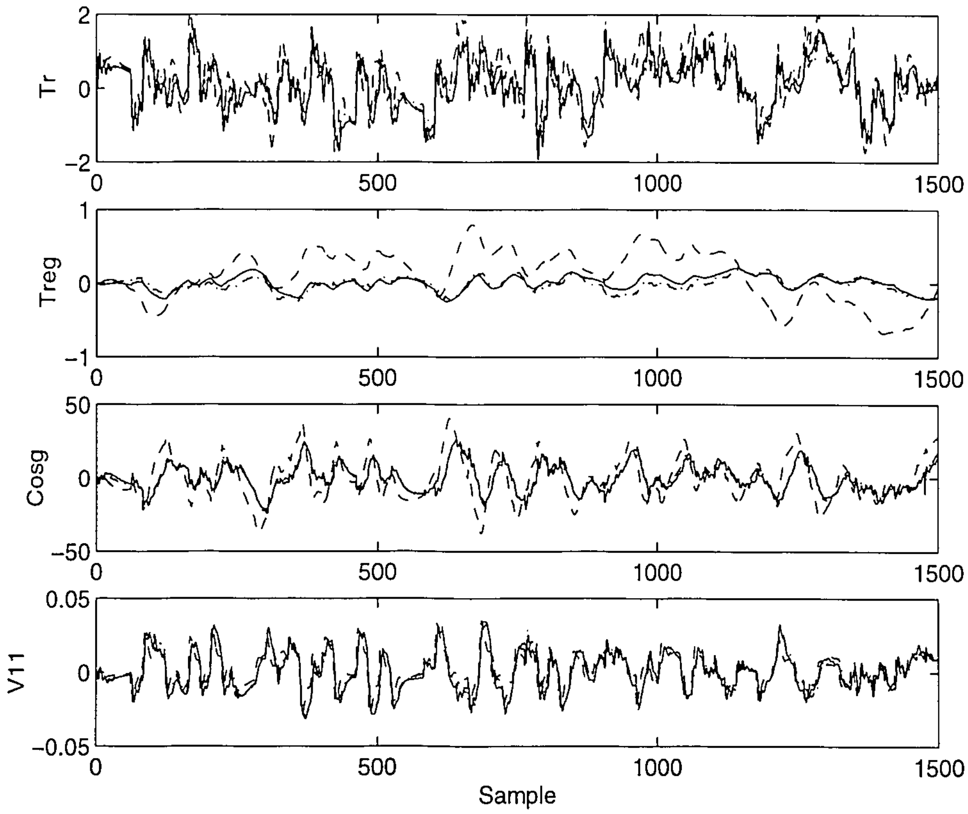


Figure 5.9: Actual and simulated output for the Amoco FCCU. Actual output (-)Solid line, prediction using subspace identification (dash-dotted), prediction using TS method (dashed)

I/O variable	Input variables					
	Feed			Slurry		
Output	K_u	w_u	k	K_u	w_u	k
T_r	-3.33	0.1	-0.349	0.833	0.002	1.355
T_{reg}	5.88	.002	.371	2.22	.0028	1.84
$O2_{sg}$	-43	.005	-.0644	-60	0.013	-0.2177
CO_{sg}	0.27	0.01	17.36	0.42	0.015	53.84
V_{11}	111	0.1	0.014	400	0.2	0.034

Table 5.6: Ultimate Gain, Ultimate frequency and Steady State Gain of the identified model obtained using subspace algorithm.

I/O variable	Input variables					
	Liftair			Diff. Pressure		
Output	K_u	w_u	k	K_u	w_u	k
T_r	6.6	0.08	1.93	-0.33	0.1	-28.21
T_{reg}	2.27	.0015	.197	-0.263	0.003	-11.23
$O2_{sg}$	-9.5	0.005	-0.043	1.0	0.012	1.02
CO_{sg}	0.04	0.006	19.41	-0.006	0.015	-330.9
V_{11}	133	0.04	.0261	-6.66	0.07	-.321

Table 5.7: Ultimate Gain, Ultimate frequency and Steady State Gain of the identified model obtained using subspace algorithm.

the models obtained from both the methods differ in the steady state gain from the actual value. This difference in the steady state gain is mostly due to the attenuation of low frequency content of the external signal. Moreover this could be also be due to the nonlinear nature of the system. The system is identified using small perturbation around the steady state value. For most of the cases the input perturbation used for identification was far less than unity. Hence a unit step response could be drive the system far away form the present operating point such that linear model is no longer accurate.

Fig 5.12 compares the frequency response of the identified model with the one obtained from open loop case. Is shows that the model obtained from subspace identification does a reasonable job in identifying high frequency part of the system. Results from subspace and two step indicates that subspace algorithm does a far better job in capturing the important dynamics of the process than two step method. Due to space limitation not all the input-output pairs are discussed here. But similar behavior was observed for other pairs too. The subspace identification tends to give better step response and frequency response fit as compared to the Two step method.

5.3 Conclusion

1. *The developed subspace algorithm accurately identifies the plant transfer function form the closed loop data.*

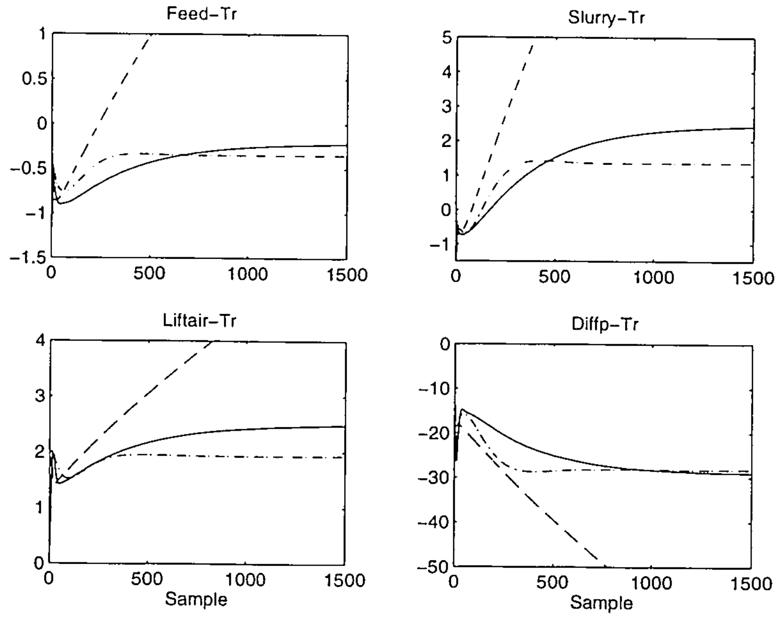


Figure 5.10: Step response of T_r for unit step change in the manipulated variable. Actual Response (solid), Subspace Identification (dash-dot), TS Method (dashed)

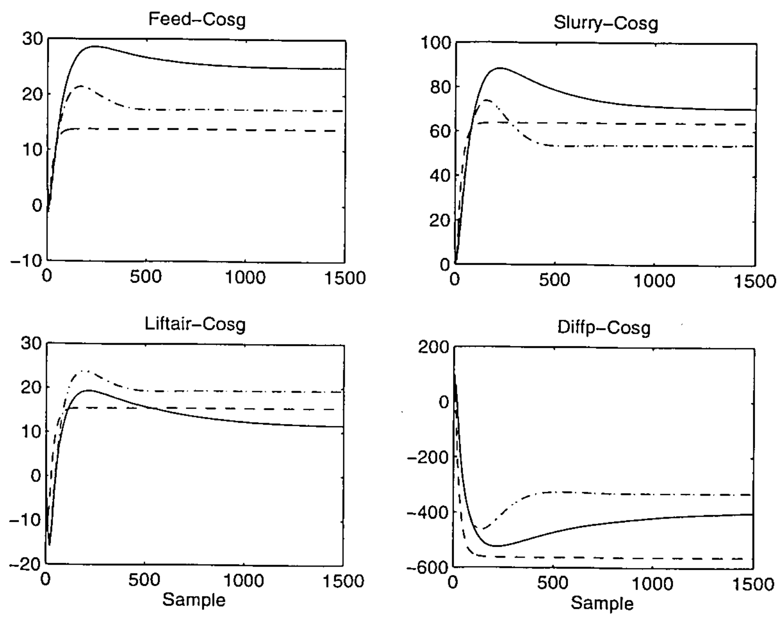


Figure 5.11: Step response of Co_{sg} for unit step change in the manipulated variable. Actual Response (solid), Subspace Identification (dash-dot), TS Method (dashed)

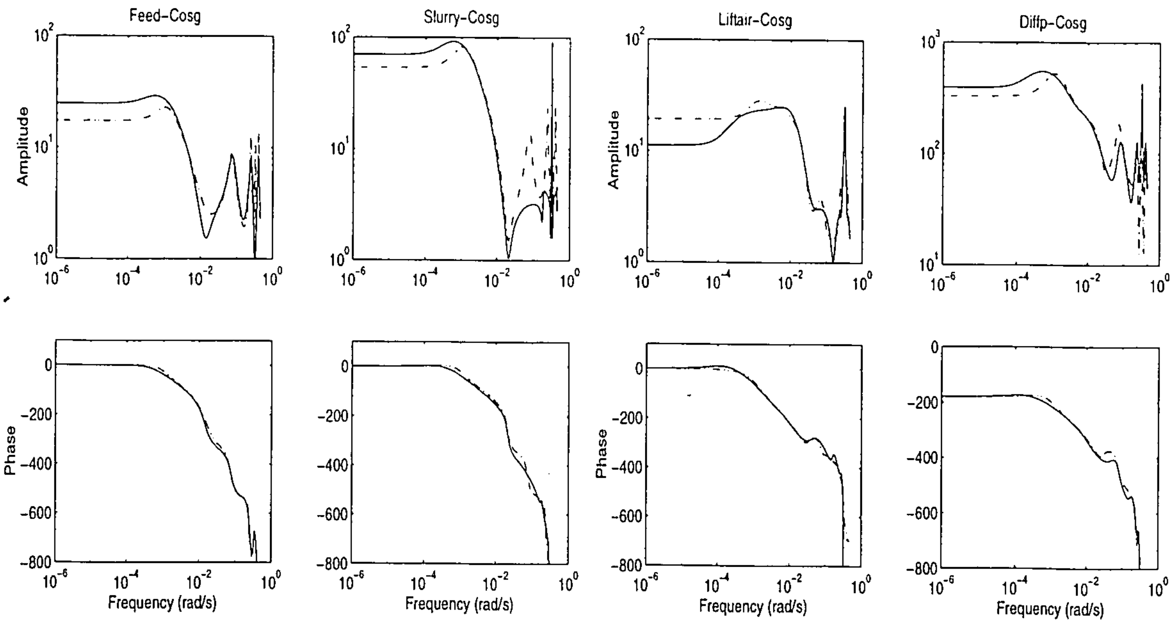


Figure 5.12: Frequency response of the identified model using Subspace Identification (dash-dot), Open loop (solid) for the output Co_{sg}

2. *Model identified using Two Step method were accurate only for few input-output pair. This suggest that the identified model parameters didn't reach their global minimum. This is due to the nonlinear nature of the optimization procedure involved during the MIMO identification.*

3. *For most of the cases the subspace model did a better job in capturing the system dynamics than Two Step method.*

Chapter 6

Summary, Conclusion and Recommendations

6.1 Conclusion

The main problem addressed in this thesis has been the identification of the process transfer function matrix for a system operating in closed-loop. The close-loop identification techniques are needed not only because of the economic and environmental constraints but also because it is helpful in the development of high performance model based controller. Since most of the process are multivariable in nature, the focus in this research project has been on the development of an identification algorithm which will perform well on MIMO systems. The problem was addressed from two different perspectives. First, studies on Prediction Error Method (PEM) were carried out to evaluate its advantages and limitations. Secondly, the subspace based state-space approach was analyzed for its potential strength and weakness. The following sections summarize the results obtained from each of the two approaches.

6.1.1 Prediction Error Method

Since a lot of result has been published in the area of prediction error methods, it was necessary to analyze each of the proposed methods in order to get a clear picture of the various identification algorithms. The insight obtained from this investigation can be summarized as follows :

1. For open-loop identification, the optimization criteria (defined by using one step ahead prediction Eq. 2.5) result in consistent estimates of process transfer function if the process and disturbance transfer functions are independently parameterized. Moreover the bias of the identified model in the frequency domain for the case of under modeling depends on the magnitude of the input spectrum. Higher values of the input spectrum at certain frequency result in lower bias of the identified model for that frequency. The bias distribution of the identified model can be effected by the use of prefilter, $L(z)$. The parameters for the independently parameterized disturbance model converge to value such that the model noise spectrum resembles the model error spectrum.

2. The basic assumption, made in the optimization criteria mentioned in the previous point, is that the plant inputs and the unmeasured disturbances are uncorrelated with one another. For the closed-loop configuration given by Fig. 2.1 this assumption is no longer valid because of the presence of the feedback term. Hence the open loop optimization criteria can not be directly applied to closed loop data. Even though the open loop optimization criteria leads to erroneous results for closed loop data, it is commonly used in practice because of its simplicity. Straightforward application of the PEM to closed loop data is also referred as direct identification. The frequency domain expression for direct identification (Eq. 2.15) indicates that the estimate of the process parameters is directly effected by the unknown disturbances. Hence the consistent estimate of the process transfer function can not be obtained even when the plant and disturbance model are independently parameterized.

3. The Two-Step method (Sec 2.3.4) divides the closed-loop problem in two open-loop problem and hence the open loop identification results can be directly applied for this case. Two-step method removes the correlation between the plant input and the plant noise by reconstructing a noise free estimate of the plant input by using the estimate of the closed loop sensitivity function. It is necessary to parameterize the plant and disturbance model independently in both the steps, to obtain good approximation of the process transfer function.

4. The input signal can be introduced at two different locations in the close-loop for the identification purpose. Simulation studies on a known linear system indicated that the controller tuning plays a vital role in deciding the location of the external signal. Since the same signal is weighted differently depending on its injection point, it is necessary to characterize the weighting function and its effect on the final bias distribution of the model. It was indicated that the signal injection location should be selected such that the corresponding closed-loop weighting function is flat in the desired frequency range. This results in a lower bias of the identified model for the frequency range of interest.

5. Simulation studies on the FCCU indicated that the TS method results in a better model as compared to DI particularly for high noise to signal ratio. This is because of the fact that TS method removes the correlation between the plant input and unmeasured disturbances. Moreover the results indicate that the bias of the identified model was dependent on the nature of the closed loop sensitivity function.

6. For the MIMO case, the Two Step method was not accurate for all the input-output transfer function. The results from 4x4 FCCU simulation suggested that not all the parameters converged to the values that correspond to the global minimum of the fitting function. Since the prediction error method solves a nonlinear optimization problem to get an estimate of the model parameters, it is difficult to guarantee the convergence of the optimization subroutine to its global minimum, particularly for the MIMO case.

6.1.2 Subspace Identification

1. Linear algebra concepts regarding matrix projection were discussed in detail. It was shown that one matrix can be projected on to another by performing certain matrix operations. The projection of one matrix on to another can be viewed as the decomposition of the row or column of one matrix in terms of other. Depending on the co-ordinate axis used during projection, the operation were characterized as orthogonal or oblique projection. It was indicated that the system properties can be revealed by geometric manipulation of certain matrices.

2. The deterministic and the stochastic algorithm for the open-loop case was discussed in the state-space framework. For deterministic algorithm it was indicated that the state sequences and the extended observability matrix can be computed through an oblique projection. The oblique projection is computed by projecting the future output Hankel matrix along the future input Hankel matrix onto the compound row space of past input-output Hankel matrix. For deterministic-stochastic case it was indicated that the deterministic algorithm has to be modified a bit to account for the stochastic component in the data.

3. Combined closed loop system dynamics for plant and controller was derived in the state-space framework. It was shown that the closed-loop problem can be reduced to an open-loop one by posing it in a joint input-output identification manner. This results in the identification of overall closed loop transfer function (consisting of plant and controller states). For the case when plant input and output were used as the augmented output vector, it was shown that performing the operation indicated by Eq. 4.20 resulted in the estimation of the plant transfer function. For the case when the controller input and output variables were used in defining the output for the joint input-output identification, the operation (Eq. 4.35) resulted in the identification of the controller transfer function. Since the controller input and output can be obtained by the algebraic manipulation of the plant input and output, it was

proved that only one identification is needed to identify both the plant and the controller transfer functions separately. It was shown that the plant, as well as controller, transfer function can be obtained separately from the overall closed-loop transfer function by performing appropriate matrix manipulations.

4. Two identification algorithms were proposed for the identification of plant and controller transfer function. Since the joint input-output identification reduces the problem to an open-loop one, the deterministic-stochastic algorithm of the open-loop case was applied to come up with the system matrices. The matrix manipulation, mentioned in the previous point, was then applied to identify the desired transfer function. Moreover, it was indicated that applying the open-loop deterministic algorithm simplifies the identification of system matrices and hence led to the reduction in the computational load. This simplified algorithm was referred as the second algorithm for the closed loop identification.

5. Simulation studies on a forth order linear system with a PI controller indicated that the algorithms identified both the process and controller model fairly accurately. The results were compared with the one obtained form direct identification and it was shown that subspace identification algorithms resulted in more accurate models. The number of block rows was one of the parameters used during identification. It was shown that increasing the number of block rows increased the accuracy of the identified model.

6. It is known that subspace algorithm tends to compute unstable models for system having lightly damped poles. The simulation study on a second order system indicated that both the proposed algorithm computed unstable models for lower numbers of block rows. A technique to guarantee the stability of identified model was proposed. It was proved that the introduction of zeros in the reversed extended controllability matrix resulted in the identification of a stable model. The results from the second order system indicated that the proposed stability modification resulted in a stable but not very accurate

model. The bias of the model identified using the stable subspace algorithm was quite high.

7. Results from the MIMO identification of the FCCU suggests that subspace algorithms resulted in consistent models for all the input-output pair, whereas for Two Step method it was found that some of the identified models were inaccurate. The model identified using subspace techniques, estimated the ultimate gain and ultimate frequency of the system pretty accurately. Moreover it was shown that the predictive nature of the model identified using subspace algorithm was better than that of model identified using Two Step method.

6.2 Recommendations

System identification is an important and rapidly growing area. Recent developments in this area have not only opened up lot of avenues to be explored but also have raised lot of questions yet to be answered. One of such promising development is the subspace based state space identification. The problem of closed loop identification of state space models has been dealt in this thesis but there are still many problems to be addressed in this area. Some of them are described in the following section.

1. Having solved the problem of closed loop identification one is equipped with the basic tool to address the problem of identifying the process model from historical data. Answer to which will be of prime industrial importance. Since most of the process industry record the daily setpoint and load disturbance changes, it will be possible to tap into the huge amount of data for system identification. The major difference in this approach from the traditional system identification techniques is that we are not injecting any external signal for identification. This could lead to some problems in the identification because the normal input may not excite all the modes of the system. Hence a guideline in the selection of data should be the first step in this direction.

Rank of the input Hankel matrix suggests the quality of the input signal and hence could be used as a basis for screening the data set.

2. Closed-loop problem has been solved from the joint input-output identification perspective. The problem is defined such that the problem reduces to an open loop one and then appropriate matrix manipulations are done to extract the information about the plant and controller separately. This procedure leads to the identification of some unobservable and uncontrollable modes in the plant and controller transfer function. These modes have to be removed by a performing minimal realization of the identified transfer function. This may not be always desirable in practice because the knowledge of tolerance limit within which all the poles and zeros are to be cancelled, is not an easy one to obtain. Hence a technique which overcomes the problem of the minimal realization step will truly be a beneficial one in the practical sense. The problem could be addressed by modifying the oblique projection and defining it for closed loop case.

3. It has been pointed out in Gevers (93) that closed loop identification is particularly useful when identified models are to be used for control. The control relevant identification is another promising area. Lot of results in the area of control relevant identification related to the PEM approach, has been published. However there are very few contributions in the area of subspace identification which address the problem of control relevant identification. The effect of weighting matrices W_1 and W_2 on the identified model has yet to be considered. It will be interesting to correlate the effect of the prefilter in PEM and the effect of weighting matrices in subspace identification on the accuracy of the identified model in the frequency domain.

4. For most of the chemical engineering plants we can classify the control structure in two major classes

   - low level regulatory controller (mostly PI and PID which are SISO in nature)

- High level model based controller (which are multivariable in nature)

It will be useful to derive the expression for closed loop transfer function for the case when the plant is controller by a constrained multivariable controller. The present derivation handles the situation when the controller can be represented in a general state space framework and hence is suitable for multiple PI or PID loops, each of which are single input and single output in nature.

5. Most of the techniques discussed in this thesis assumes that the plant and controller transfer function is linear in nature. The model predictive controller is no longer linear in nature when it is operating under constraints. Hence the assumption of a linear controller model during identification could lead to an inaccurate model. Incorporation of the constraints during identification is a challenging area and still is an open problem. Moreover, the problem of input signal design has to be revisited if the inputs to the plant are constrained.

6. The techniques described in this thesis assumes that we have infinite amount of data at out disposal (the number of columns used in the definition of input and output Hankel matrices tends to infinity). The results indicated here refers to the asymptotic unbiasedness of the algorithms. This however doesn't give us any indication about the accuracy of the identification algorithm for finite data length. Statistical analysis of the subspace algorithm will provide us with the necessary tools required to evaluate the performance of subspace algorithm for finite data. Some results on asymptotic statistical distribution of subspace algorithms have been recently reported in the literature. (Viberg et al., 1991; Viberg M. and Ljung, 1993; Ottersten and Viberg, 1994; Ottersten et al., 1992)

7. For the case where the data is of finite length, the determination of the order of the system is not an easy task. For infinite measurements, the number of singular values different form zero reveals the order of the system but for finite set of measurements one has to look for the gap in the singular value spectrum.

A statistical test in that direction could be developed to distinguish between the relevant and irrelevant (noise or unmeasured disturbances) singular values.

8. The subspace techniques described in this thesis primarily focus on the identification of a black box model. However, sometimes information like rise time, steady state gain, stability etc. are known in advance. The modification to incorporate these information in the subspace identification algorithm will greatly enhance there use in practice. The technique to incorporate stability requirement during the identification has been considered in this thesis. Similar results has been proposed in (Maciejowski, 1995).

9. One of the main application of closed identification can be to retune the existing model predictive controller. Since the developed methodologies can be applied in closed loop, there is no need to switch off the controller for the identification experiment. A guideline as when the re-identification of the process is needed, could be beneficial from the industrial perspective. Hence a mathematical base for evaluation of controller performance and the subsequent identification phase is an interesting area to be explored.

10. The major focus of the closed-loop algorithms developed in this thesis is in the identification of the deterministic component of the system. But most of the real life processes are effected by unknown disturbances or noise. Hence the stochastic problem is also a vital component of any approach. The stochastic problem addressed in this research assumes that the disturbances effecting the plant are zero mean and stationary in nature. This could be a very restrictive assumption and may not be always fulfilled in practice. The extension of the present algorithm for non-stationary disturbances (non-zero mean) will be a useful development.

Bibliography

Akaike, H. (1974). Stochastic theory of minimal realization. *IEEE Transactions on Automatic Control*, 19:667–674.

Akaike, H. (1975). Markovian representation of stochastic process by canonical variables. *Siam J. Control*, 13(1):162–173.

Aling, H. and Bosgra, O. H. (1990). Structural identifiability conditions for system operating in closed loop. *Realization and Modeling in System Theory.*

Anderson, B. and Gevers, M. (1979). Identifiability of closed loop systems using the joint input-output identification method. *Proc. of the 5th IFAC Symposium on Identificaiton and System Parameter Estimation*, pages 645–652.

Anderson, B. and Gevers, M. (1982). Identifiability of linear stochastic systems operating under linear feedback. *Automatica*, 18(2):195–203.

Arun, K. and Kung, S. (1990). Balanced approximation of stochastic systems. *SIAM Journal. of Matrix Analysis and Application*, 11:42–68.

Astrom, K. (1993). Matching criteria for control and identification. In *Proc. of the European Control Conference.*, pages 248–251, Groningen, The Netherlands.

Astrom, K. . J. and Wittenmark, B. (1989). *Adaptive Control.* Addison-Wesleyc, Reading, MA.

Bayard, D. (1992). An algorithm for state-space frequency domain identification without window distortions. In *Proc. 31st Conf. on Decision and Control*, pages 1707–1712, Tucson, AZ.

De Moor, B. (1988). *Mathematical Concepts and Techniques for Modeling of Static and Dyanmical System.* PhD thesis, Kath. Universiteit Leuven, Heverlee, Belgium.

De Moor, B., Vandewalle, J., Moonen, M., and Van Mieghem, P. (1988). A geometric approach for identification of state-space models with singular value decomposition. In *Proc IFAC 88*, pages 700–704, Beijing, China.

Desai, U., Kirkpatrick, R., and Pal, D. (1985). A realization approach to stochastic model reduction. *Inter. Journal of Control*, 42(4):821–838.

Desai, U. and Pal, D. (1984). A transformation approach to stochastic model reduction. *IEEE Trans. on Automatic Control*, 29(12).

Dicksinson, B., Morf, M., and Kailath, T. (1974a). Canonical matrix fraction and state space description for deterministic and stochastic lineat systems. *IEEE Transactions on Automatic Control*, AC-19(1):656–667.

Dicksinson, B., Morf, M., and Kailath, T. (1974b). A minimal realization algorithm for matrix sequences. *IEEE Transactions on Automatic Control*, AC-19(1):31–38.

Gevers, M. (1978). On the identification of feedback systems. In *Proc. of the 4th IFAC symposium on Identification and System Parameter Estimation*, pages 1621–1630, Tbilisi, USSR.

Gevers, M. (1993). Towards a joint design of identification and control ? In *Essays on Control: Prespectives in the Theory and its Application*, pages 111–151. Trentelman, H.L and Willems, J.C (Eds.), Birkhauser, Boston.

Gevers, M. and Ljung, L. (1985). Benifits of feedback in experimental design. *IFAC Identification and System Parameter Estimation, UK*, pages 909–914.

Gevers, M. and Ljung, L. (1986). Optimal experiment design with respect to intended model application. *Automatica*, 22(5):543–554.

Goodwin, G. C. and Payne, R. L. (1977). *Dynamic System Identification: Experiment Design and Data Analysis.* Academic Press, New York.

Hansen, F., Franklin, G., and Kosut, R. (1989). Closed-loop identification via the fractional representation: Experiment design. *American Control Conference, Pittsburgh*, pages 1422–1427.

Heuberger, P. (1991). *On Approxiamte System Identifiaction with System Based Orthonormal Functions.* PhD thesis, Delft University of Technology, the Netherlands.

Ho, B. and Kalman, R. (1966). Efficient construction of linear state variable models from input/output functions. *Regelungstechnik.*

Jansson, M. and Wahlberg, B. (1994). N4sid linear regression. *Proc. Conf. on Decision and COntrol.* Submitted for publication.

Jha, H. and Georgakis, C. (1996). Subspace identification algorithm to identify state space model from closed loop data. *CPMC Report 22A.*

Jha, H., Sistu, P., and Georgakis, C. (1995). Control-oriented closed loop identification. Technical Report 21, CPMC, Lehigh University.

King, A., Desai, U., and Skelton, R. (1988). A generalized approach to q-markov covariance equivalent realizations of discrete systems,. *Automatica*, 24(4):507–515.

Kung, S. (1978). A new identification and model reduction algorithm via singular value decomposition. In *Proceedings 12th Asilomar Conf on Circuits, Systems, and Computers*, pages 705–714.

Larimore, W. (1990). Canonical variate analysis in identification, filtering and adaptive control. *Proc. 29th Conference on Decision and Control*, pages 596–604.

Liu, K. and Skelton, R. (1991). Identification and control of nasa's aces structure. *Proc. of American Control Conf.*, pages 3000–3006.

Ljung, L. (1987). *System Identification: Theory for the User.* Prentice-Hall.

Ljung, L. (1991). A simple start-up procedure for canonical form state space identification based on subspace approximation. In *Proceedings CDC91*, pages 1333–1336, Brighton, UK.

Ljung, L., Gustavsson, I., and Soderstrom, T. (1974). Identification of linear multivariable system operating under linear feedback control. *IEEE Trans. Autom. Control.*, 19(6):836–840.

Maciejowski, J. (1995). Guaranteed stability with subspace methods. *System and Controls Letters*, 26:153–156.

McFarlane, R., Reineman, R. C., Bartee, J., and Georgakis, C. (1993). Dynamic simulator for a model iv fluid catalytic cracking unit. *Comp. Chem. Engg*, 17:275.

McFarlane, R. and Rivera, D. (1992). Identification of distillation systems. In *Practical Distillation Control*, chapter 7. Van Nostrand Reinhold.

McKelvey, T. (1995a). Frequency domain system identification with iv based subspace algorithm. Technical Report LiTH-ISY-R-1775, Dept of EE, Linkoping University, Sweden. Submitted to 34th CDC.

McKelvey, T. (1995b). Frequency weighted subspace based system identification in frequency domain. Technical Report LiTH-ISY-R-1776, Dept of EE, Linkoping University, Sweden. Submitted to 34th CDC.

McKelvey, T., Akcay, H., and Ljung, L. (1994). Efficient construction of transfer function form frequency response data. Technical Report LiTH-ISY-R-1609, Dept of EE, Linkoping University, Sweden.

Mehra, R. (1981). *Choice of input signals. In Trends and Progress in System Identification.* (P.Eykhoff Ed.), Pergamon Press, Elmsford, N.Y.

Moonen, M., DeMoor, B., Vandenberghe, L., and Vanderwalle, J. (1989). On and off line identification of linear state-space model. *Int. J. Control*, 49(1):219–232.

Moonen, M. and Vanderwalle, J. (1990). A qsvd approach to on and off line state space identification. *Int. J. Control*, 51(5):1133–1146.

Morari, M. and Zafiriou, E. (1989). *Robust Process Control.* Prentice-Hall, Englewood Cliffs, NJ.

Ottersten, B. and Viberg, M. (1994). A subspace based instrumental variable method for state space system identification. In *Proc. of SYSID '94*, volume 2, pages 139–144, Copenhagen, Denmark.

Ottersten, B., Viberg, M., and Kailath, M. (1992). Analysis of subspace fitting and ml techniques for parameter estimation from sensor array data. *IEEE trans. on Signal Processing*, 40(3):590–600.

Rao, B. and Arun, K. (1992). Model based processing of signals: A state space approach. In *Proceedings of IEEE*, volume 80, pages 283–309.

Rivera, D.E., A. T. and Bhatnagar, S. (1992). Control-relevant methodology for closed-loop identification. *AICHE National Meeting, Miami Beach, Florida.*

Rivera, D., Pollard, J., and Garcia, C. (1992). Control-relevant prefiltering: a systematic design approach and case study. *IEEE Trans. Auto. Contr.*, 37(7):964.

Schrama, R. (1991). An open-loop solution to the approximate closed-loop identification problem. In *Proc 9th IFAC/IFORS Symposium on Identification and System Parameter Estimation*, pages 1602–1607, Budapest, Hungry.

Schrama, R. (1992). Accurate identification and control; the necessity of an iterative scheme. *IEEE Trans. Autom. Control.*, 37(7):991–994.

Shook, D., Mohtadi, C., and Shah, S. (1992). A control-relevant identification strategy for gpc. *IEEE Trans. Autom. Control.*, 37(7):975–980.

Sin, K. and Goodwin, G. (1980). Checkable conditions for identifiability of linear system operating in closed loop. *IEEE Trans. Autom. Control*, 25(4):722–729.

Soderstrom, T., Gustavsson, I., and Ljung, L. (1975). Identifiability conditions for linear systems operating in closed loop. *Int. J. Control*, 21(2):243–255.

Soderstrom, T. and Stoica, P. (1989). *System Identification.* Prentice-Hall.

Tulken, H. (1990). Generalized binary noise test -signal concept for improved identification - experiment design. *Automatica*, 26(1):37.

Van der Klauw, Verhaegen, M. and den Bosch P.P.J, V. (1991). State space identification of closed loop system. *Proc. IEEE Conference on Decision and Control*, pages 1327–1332.

Van Overschee, P. (1995). *Subspace Identification, Theory - Implementation - Application.* PhD thesis, Department of Electrical Engg., Katholieke Universiteit Leuven.

Van Overschee, P. and De Moor, B. (1992). Two subspace algorithm for the identification of combined deterministic stochastic systems. In *31st IEEE Conference on Decision and Control*, pages 511–516.

Van Overschee, P. and De Moor, B. (1993a). N4sid numerical algorithms for state space subspace system identification. In *Proc. of the World Congress of International Federation of Automatic Control, IFAC*, volume 7, pages 361–364, Sydney, Australia.

Van Overschee, P. and De Moor, B. (1993b). A unifying theorem for three subspace system identification algorithms. Technical Report ESAT-SISTA Report 93-50I, Department of Electriacal Engg, Katholieke Universiteit Leuven.

Van Overschee, P. and De Moor, B. (1994a). About the choice of state space bases in combined deterministic-stochastic subspace identification. Technical Report ESAT-SISTA Report 94-24I, Department of Electrical Engg, Katholieke Universiteit Leuven.

Van Overschee, P. and De Moor, B. (1994b). N4sid: Subspace algorithm for the identification of combined deterministic-stochasitc system. *Automatica, Special Issue on Statistical Signal Processing and Control*, 30(1):75–93.

Van Overschee, P. and De Moor, B. (1994c). A unifying theorem for three subspace system identification algorithms. In *Proc. of SYSID'94*, volume 2, pages 145–150, Copenhagen, Denmark.

Van Overschee, P. and De Moor, B. (1994d). A unifying theorem for three subspace system identification algorithms. *American Control Conference*, pages 1645–1649.

VanDenHof, P. and Schrama, R. (1993). An indirect method for transfer function estimation from closed-loop data. *Automatica*, 29(7):1523–1527.

VanDer Veen, A., Deprettere, E., and Swindlehurst, A. (1993). Subspace-based signal analysis using singular value decomposition. *Proceedings of IEEE*, 81(9):1277–1308.

Verhaegen, M. (1991). A novel non-iterative mimo state space model identification technique. In *Proc 9th IFAC/IFORS Symposium on Identification and System Parameter Estimation*, pages 1349–1354, Budapest, Hungry.

Verhaegen, M. (1993). Subspace model identification. part iii : analysis of the ordinary output-error state space model identification algorithm. *Int. Journal of Control*, 58:555–586.

Verhaegen, M. (1994). Identification of the deterministic part of mimo state space models given in innovations from the input-output data. *Automatica, Special Issue on Statictical Signal Processing and Control*, 30(1):61–74.

Verhaegen, M. and Deprettere, E. (1991). A fast, recursive mimo state space model identification algorithm. In *Proc CDC*, Brighton, England.

Verhaegen, M. and Dewilde, P. (1992). Subspace model identification. part i: The output-error state space model identification class of algorithm. *Int. Journal of Control*, 56:1187–1210.

Viberg, M. (1994). Subspace methods in system identification. In *Proc. of SYSID'94*, volume 1, pages 1–12, Copenhagen, Denmark.

Viberg, M., Ottersten, B., Wahlberg, B., and Ljung, L. (1991). A statistical perspective on state-space modeling using subspace methods. In *Proc. of the 30th IEEE Conference on Decision and Control*, volume 2, pages 1337–1342, Brighton, England.

Viberg M., B. Ottersten, B. W. and Ljung, L. (1993). Performance of subspace based state space system identification methods. In *Proc. of the 12th IFAC '93*, volume 7, pages 369–372, Sydney, Australia.

Wellstead, P. (1978). The identification and parameter estimation of feedback systems. In *Proc. of the 4th IFAC Symposium on identification and system parameter estimation*, pages 1593–1600, Tbilisi, USSR.

Zarrop, M. B. (1979). *Optimal Experimental Design for Dynamic System Identification*. Springer-Verlang, New York.

Zeiger, H. and McEwen, A. (1974). Approximate linear realization of given dimension via ho's algorithm. *IEEE Transactions on Automatic Control*, 19:153.

Zeigler, J. and Nichols, N. B. (1942). Optimum settings for automatic controllers. *Trans American Society Mechanical Engg.*, 64:759–768.

Zheng, W. and .C, F. (1991). A new look at least-squares identification of closed-loop systems. In *Preprints of the 9th IFAC Symposium on Identification and System Parameter Estimation*, pages 1374–1378, Budapest, Hungary.

Appendix A

Overall Closed Loop State-Space Model

From Fig. 5.1 we have

$$\begin{aligned}
u &= u_2 + y_c \\
u_c &= u_1 - y \\
&= u_1 - C_p x_p - D_p u - v_k \\
u_c &= K u_1 - K C_p x_p - K D_p u_2 - K D_p C_c x_c - K v_k \quad , \quad K = (I + D_p D_c)^{-1}
\end{aligned} \tag{A.1}$$

Similarly process input u can be calculated as

$$\begin{aligned}
u &= u_2 + \overbrace{C_c x_c + D_c(K u_1 - K C_p x_p - K D_p u_2 - K D_p C_c x_c - K v_k}^{y_c} \\
&= (C_c - D_c K D_p C_c) x_c - D_c K C_p x_p + D_c K u_1 + (I - K D_p) u_2 - D_c K v_k
\end{aligned} \tag{A.2}$$

The controller and plant states can be written as

$$\begin{aligned}
x_c^{k+1} &= A_c x_c^k + B_c u_c \\
&= -B_c K C_p x_p + (A_c - B_c K D_p C_c) x_c + B_c K u_1 - B_c K D_p u_2 - B_c K v_k \\
\\
x_p^{k+1} &= A_p x_p^k + B_p u + w_k \\
&= (A_p - B_p D_c K C_p) x_p + B_p (C_c - D_c K D_p C_c) x_c + B_p D_c K u_1 \\
&\quad + (B_p - B_p K D_p) u_2 - B_p D_c K v_k + w_k
\end{aligned} \tag{A.3}$$

Plant input u and plant output y can be written in terms of external signal u_1 and u_2 as

$$\begin{aligned} y = \;& C_p x_p(k) + D_p u(k) + v_k \\ = \;& (C_p + D_p D_c K C_c) x_p + D_p (C_c - D_c K D_p C_c) x_c \\ & + D_p D_c K u_1 + D_p (I - K D_p) u_2 - D_p D_c K v_k + v_k \\ u = \;& (C_c - D_c K D_p C_c) x_c - D_c K C_p x_p \\ & + D_c K u_1 + (I - K D_p) u_2 - D_c K v_k \end{aligned} \tag{A.4}$$

Combining Equation we can write

$$\begin{aligned} \begin{bmatrix} x_p^{k+1} \\ x_c^{k+1} \end{bmatrix} &= \begin{bmatrix} A_p - B_p D_c K C_p & B_p (C_c - D_c K D_p C_c) \\ -B_c K C_p & A_c K D_p C_c \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} \\ &+ \begin{bmatrix} B_p D_c K & B_p - B_p K D_p \\ B_c K & -B_c K D_p \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} I & -B_p D_c K \\ 0 & -B_c K \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \end{aligned} \tag{A.5}$$

$$\begin{aligned} \begin{bmatrix} u \\ y \end{bmatrix} &= \begin{bmatrix} -D_c K C_p & C_c - D_c K D_p C_c \\ C_p - D_p D_c K C_c & D_p (C_c - D_c K D_p C_c) \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} \\ &+ \begin{bmatrix} D_c K & I - K D_p \\ D_p D_c K & D_p (I - K D_p)) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & -D_p D_c K + I \\ 0 & -D_c K \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \end{aligned} \tag{A.6}$$

More over manipulating the above set of equation such that the combined output consist of controller input and output we have.

$$\begin{aligned} \begin{bmatrix} y_c \\ u_c \end{bmatrix} &= \begin{bmatrix} u - u_2 \\ u_1 - u \end{bmatrix} \\ &= \begin{bmatrix} -D_c K C_p & C_c - D_c K D_p C_c \\ -C_p + D_p D_c K C_c & D_p (C_c - D_c K D_p C_c) \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} \\ &+ \begin{bmatrix} D_c K & -K D_p \\ I - D_p D_c K & -D_p (I - K D_p)) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & -D_p D_c K + I \\ 0 & D_c K \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \end{aligned} \tag{A.7}$$

Since for most of the cases of interest D_p (Plant feed through term is zero) the overall closed loop equation for plant and controller reduces to

$$\begin{bmatrix} x_p^{k+1} \\ x_c^{k+1} \end{bmatrix} = \begin{bmatrix} A_p - B_pD_cC_p & B_pC_c \\ -B_cC_p & A_c \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} B_pD_c & B_p \\ B_c & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} I & -B_pD_c \\ 0 & -B_c \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{A.8}$$

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} -D_cC_p & C_c \\ C_p & 0 \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} D_c & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & I \\ 0 & -D_c \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{A.9}$$

$$\begin{bmatrix} y_c \\ u_c \end{bmatrix} = \begin{bmatrix} u - u_2 \\ u_1 - y \end{bmatrix} = \begin{bmatrix} -D_cC_p & C_c \\ -C_p & 0 \end{bmatrix} \begin{bmatrix} x_p^k \\ x_c^k \end{bmatrix} + \begin{bmatrix} D_c & 0 \\ I & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & I \\ 0 & -D_c \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix} \tag{A.10}$$

Appendix B

Identification of Plant Transfer Function

Proof: We have

$$A'' = A' - B_2' {D_{12}'}^{-1} C_1' \qquad B'' = B_2' {D_{12}'}^{-1} \qquad C'' = C_2' - D_{22}' {D_{12}'}^{-1} C_1' \qquad D'' = D_{22}' {D_{12}'}^{-1}$$

whereas as calculated before

$$A' = T \begin{bmatrix} A_p - B_p D_c C_p & B_p C_c \\ -B_c C_p & A_c \end{bmatrix} T^{-1}, \qquad B' = T \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \qquad \begin{array}{l} C_1' = [-D_c C_p \;\; C_c]\, T^{-1} \\ C_2' = [C_p \;\; 0]\, T^{-1} \end{array}$$

$$D_{22}' = 0, \qquad D_{12}' = I, \qquad \text{with} \qquad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

Defining $S = T^{-1}$ it can be shown that

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} (T_{11} - T_{21} T_{11}^{-1} T_{12})^{-1} & -T_{22} T_{21} (T_{11} - T_{12} T_{22}^{-1} T_{21})^{-1} \\ -T_{11} T_{12} (T_{22} - T_{21} T_{11}^{-1} T_{12})^{-1} & (T_{22} - T_{21} T_{11}^{-1} T_{12})^{-1} \end{bmatrix}$$

Substituting and simplifying we get

$$A'' = \begin{bmatrix} (T_{11} A_p - T_{12} B_c C_p) S_{11} + T_{12} A_c S_{21} & (T_{11} A_p - T_{12} B_c C_p) S_{12} + T_{12} A_c S_{22} \\ (T_{21} A_p - T_{22} B_c C_p) S_{11} + T_{22} A_c S_{21} & (T_{21} A_p - T_{22} B_c C_p) S_{12} + T_{22} A_c S_{22} \end{bmatrix}$$

$$B'' = \begin{bmatrix} T_{11}B_p \\ T_{21}B_p \end{bmatrix}, \qquad C'' = [C_pS_{11}C_pS_{12}], \qquad D'' = 0$$

Applying the Similarity transformation $\mathcal{M}$ we get

$$\mathcal{M} = \begin{bmatrix} I & S_{11}^{-1}S_{12} \\ 0 & -S_{11}^{-1} \end{bmatrix} \qquad \mathcal{M}^{-1} = \begin{bmatrix} I & S_{12} \\ 0 & -S_{11} \end{bmatrix}$$

$$A'' = \begin{bmatrix} S_{11}^{-1}A_pS_{11} & 0 \\ -S_{11}((T_{21}A_p - T_{22}B_cC_p) + T_{22}A_cS_{21} & -S_{11}T_{22}A_c(S_{21}S_{12} - S_{22}S_{11}) \end{bmatrix}$$

$$B'' = \begin{bmatrix} S_{11}^{-1}B_p \\ -S_{11}T_{21}B_p \end{bmatrix}, \qquad C'' = [C_pS_{11} \quad 0], \qquad D'' = 0;$$

But the observable and controllable part of the system is

$$A = S_{11}^{-1}A_pS_{11} \qquad B = S_{11}^{-1}B_P \qquad C = C_pS_{11}, \qquad D = 0$$

hence the identified system is just a similarity transform of the actual system matrices.

Appendix C

Identification of Controller transfer function

Proof: We have

$$A'' = A' + B_1'(I - D_{21}')^{-1}C_2' \qquad B'' = B_1'(I - D_{21}')^{-1}$$
$$C'' = C_1' + D_{11}'(I - D_{21}')^{-1}C_2' \qquad D'' = D_{22}'(I - D_{21}')^{-1}$$

whereas as calculated before

$$A' = T\begin{bmatrix} A_p - B_pD_cC_p & B_pC_c \\ -B_cC_p & A_c \end{bmatrix}T^{-1}, \qquad B_1' = T\begin{bmatrix} B_pD_c \\ B_c \end{bmatrix}, \qquad \begin{array}{l} C_1' = [-D_cC_p \ \ C_c]\,T^{-1} \\ C_2' = [C_p \ \ 0]\,T^{-1} \end{array}$$

$$D_{11}' = D_c, \qquad D_{21}' = 0, \qquad \text{with} \qquad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

Defining $S = T^{-1}$ it can be shown that

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} (T_{11} - T_{21}T_{11}^{-1}T_{12})^{-1} & -T_{22}T_{21}(T_{11} - T_{12}T_{22}^{-1}T_{21})^{-1} \\ -T_{11}T_{12}(T_{22} - T_{21}T_{11}^{-1}T_{12})^{-1} & (T_{22} - T_{21}T_{11}^{-1}T_{12})^{-1} \end{bmatrix}$$

Substituting and simplifying we get

$$A'' = \begin{bmatrix} T_{11}A_pS_{11} + (T_{11}B_pC_c + T_{12}A_c)S_{21} & T_{11}A_pS_{12} + (T_{11}B_pC_c + T_{12}A_c)S_{22} \\ T_{21}A_pS_{11} + (T_{21}B_pC_c + T_{22}A_c)S_{21} & T_{21}A_pS_{12} + (T_{21}B_pC_c + T_{22}A_c)S_{22} \end{bmatrix}$$

$$B'' = \begin{bmatrix} T_{11}B_pD_c + T_{12}B_c \\ T_{21}B_pD_c + T_{22}B_c \end{bmatrix}, \qquad C'' = [C_cS_{21}C_cS_{22}], \qquad D'' = D_c$$

Applying the Similarity transformation $\mathcal{M}$ we get

$$\mathcal{M} = \begin{bmatrix} I & 0 \\ S_{21} & S_{22} \end{bmatrix} \quad \mathcal{M}^{-1} = \begin{bmatrix} I & 0 \\ -S_{22}^{-1}S_{21} & -S_{22}^{-1} \end{bmatrix}$$

$$A'' = \begin{bmatrix} T_{11}A_p(S_{11} - S_{12}S_{22}^{-1}S_{21}) & T_{11}A_pS_{12}S_{22}^{-1} + T_{11}B_pC_c + T_{12}A_c \\ 0 & A_c \end{bmatrix}$$

$$B'' = \begin{bmatrix} T_{11}B_pC_c + T_{12}B_c \\ B_c \end{bmatrix}, \qquad C'' = [0 \quad C_c], \qquad D'' = D_c;$$

But the observable and controllable part of the system is

$$A = A_c \qquad B = B_c \qquad C = C_c, \qquad D = D_c$$

Appendix D

Notations

Notation	Description
A, B, C, D	Dynamical System Matrices
A_p, B_p, C_p, D_p	Plant system matrices
A_c, B_c, C_c, D_c	Controller system matrices
$G(z)$	Deterministic transfer function $D + C(zI - A)^{-1}B$
G_p	Plant transfer function $D_p + C_p(zI - A_p)^{-1}B_p$
G_c	Controller transfer function $D_c + C_c(zI - A_c)^{-1}B_c$
H_i^d	Toeplitz matrix containing the
	deterministic Markov parameter $D, CB, CAB, \cdots$
i	Number of block rows in block Hankel Matrix
I_n	Identity Matrix ($ntimes\ n$)
j	Number of block column in block Hankel Matrix
l	Number of outputs
m	Number of inputs
Θ_i	Oblique Projection $Y_{i\|2i-1}/_{U_{i\|2i-1}}W_{o\|i}$
Q, R, S	Covariance and cross-covariance matrices of
	the measurement and process noise
T	Non singular $n \times n$ similarity transformation
u_k	Input at time instant k

U, S, V	Matrices of singular value decomposition
$U_{0\|i-1}$	Input block Hankel Matrix. Subscript indicates the indices of the first and last row of the matrix
U_p, U_p^+	Past Inputs, $U_{0\|i-1}, U_{0\|i}$ respectively
U_f, U_f^+	Future Inputs, $U_{i\|2i-1}, U_{i+1\|2i-1}$ respectively
v_k, w_k	Measurement and Process noise respectively
$W_{0\|i-1}$	Past Input ($U_{0\|i-1}$)-Output ($Y_{0\|i-1}$) block Hankel Matrix.
W_p, W_p^+	Past Inputs, (U_p, U_p^+) and outputs (Y_p, Y_p^+)
x_k	States at time instant k
X_i	State Sequence. The subscript indicates the index of the first element
X_p, X_f	Past and Future state sequences
y_k	Output at time instant k
$Y_{0\|i-1}$	Output block Hankel Matrix. Subscript indicates the indices of the first and last row of the matrix
Y_p, Y_p^+	Past Outputs, $Y_{0\|i-1}, Y_{0\|i}$ respectively
Y_f, Y_f^+	Future Inputs, $Y_{i\|2i-1}, Y_{i+1\|2i-1}$ respectively
Γ_i	Extended observability matrix
$\underline{\Gamma_i}$	Extended observability matrix Γ_i, without the last l rows
$\overline{\Gamma_i}$	Extended observability matrix Γ_i, without the first l rows
δ_{pq}	Kronecker Delta
Δ	Delay operator
Δ_i^d	Reversed extended controllability matrix of {A,B}
Π_A	Operator projecting the row space of a matrix onto the row space of A
$\Pi_{A^\perp}$	Operator projecting the row space of a matrix onto the orthogonal complement of the row space of A
$A^\dagger$	Pseudo-inverse of A.
$\Re_l$	Vector Space of l-dimensional vectors
$\Re_{l\times m}$	Vector Space of $l \times m$ -dimensional vectors

A/B	Projection of Row space of A on row space to B
$A/\begin{pmatrix} B \\ C \end{pmatrix}$	Projection of Row space of A on the combined row space of B and C
$A/_B C$	Projection of Row space of A along the row space of B on row space of C

Abbreviations

ARX	Auto-Regressive with eXogenous input
CVA	Canonical Variate Analysis
DI	Direct Identification
FCCU	Fluid Catalytic Cracking Unit
FIR	Finite Impulse Response
GBN	Generalized Binary Noise
II	Indirect Identification
IV	Instrumental Variable Method
JIO	Joint Input-Output Identification
MOESP	Multivariable Output-Error State space
MIMO	Multi-input multi-output
MISO	Multi-input single-output
N4SID	Numerical Algorithms for Subspace State Space System Identification
OE	Output Error
PEM	Prediction Error Method
PID	Proportional Integral Differential
PRBS	Pseudo-Random Binary Signal
SISO	Single-input single-output
SVD	Singular Value Decomposition
TS	Two-Step Method

Vita - Hemant Jha

Date of Birth	Sept 18th, 1972
Place of Birth	India
Parents	Shreekant and Prema Jha
Education	Bachelor of Science, Chemical Engg (1993) Indian Institute of Technology Kharagpur, India
Experience	
1996-Present	Chemical Engineer Dynamic Matrix Control Corporation
1993-94	Junior Engineer Essar Steel, India
1992	Summer Intern National Chemical Lab, Pune, India

END OF TITLE